Deformation properties of fine-grained soils from seismic tests

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ABSTRACT: Geotechnical design requires the prediction of soil structure interaction, for which the deformation properties of the soil are needed. Little guidance can be found in the literature for estimating the soil modulus during undrained loading. Therefore, over-simplified methods are frequently used even for the analysis of complex problems. The concepts used to describe the deformation behavior of fine-grained, normally consolidated soils are presented and critically reviewed. The deformation properties (shear modulus) at small and large strain are discussed. Based on a comprehensive survey of seismic field and laboratory data, it is possible to predict the shear modulus at small strain and the variation of the shear modulus with increasing shear strain. A relationship is proposed which can be used to predict the variation of the normalized shear modulus as a function of shear strain. It can be shown that the strain rate at seismic small-strain testing is slow and comparable to that of conventional geotechnical laboratory tests. The starting point of the stress-strain curve (at low shear strain level) can be accurately established from seismic tests, and its end point (at high strain) by conventional shear tests. The variation of the shear modulus with strain can be determined from resonant column tests. A numerical model is presented which makes it possible to predict the variation of shear modulus as a function of shear strain. The practical application of the concept is illustrated by a case history, where good agreement was obtained between predicted and measured deformation properties.

1 INTRODUCTION

1.1 *International Conference on Geotechnical Site Characterization, ISC in 1998*

The prediction of the deformation behavior of soils has been an important task in geotechnical research and has become increasingly important as more sophisticated analytical methods have become available. At the first International Conference on Geotechnical Site Characterization, ISC in 1998, several papers were presented, which addressed this topic. In this context, geophysical testing – and in particular seismic testing – can play an important role. Many valuable concepts were presented in one of the Theme Lectures "Deformation and in situ stress measurements", Fahey (1998). The paper outlined a generally applicable framework for establishing deformation parameters, which are required for deformation analysis of geotechnical structures. In his conclusions, Fahey stated that: "*predictions of soil deformations under the influence of foundation loads have been generally found to be of very limited accuracy. A major reason for this has been that the non-linearity of the stress-strain response in this strain range has not been taken into account until recently*". He concluded that "*a number of questions needed to be answered with the seismic methods, particularly how the effect of soil "fabric" on shear wave velocity can be differentiated from the effects of various principle stresses*".

The present paper addresses the same issues, but with emphasis on the undrained deformation behavior of normally consolidated fine-grained soils. The objective is to present a practically applicable concept for establishing the stress-strain behavior from very low to large strains. Although the paper focuses on normally consolidated soils at undrained conditions, the basic concept presented herein could be expanded to other soil types and loading conditions.

1.2 *Static versus Dynamic Soil Behavior*

Research on the stress-strain behavior of soils has been an important issue in earthquake and off-shore engineering. Major progress has been made in developing laboratory and field testing methods which have become routine tools for practicing engineers. Today it is possible to solve even complex dynamic soil-structure interaction problems. However, these advances have not been recognized by geotechnical engineers, and surprisingly crude soil models are still being used for analyzing static soil-structure interaction problems. One reason for this gap of knowledge between soil dynamics and traditional geotechnical engineering was – and in many cases still is – the notion that dynamic (and cyclic) soil properties can not be used for the analysis of static geotechnical problems.

This paper aims to demonstrate that the rate of loading during seismic small-strain testing is comparable to - or even slower than – most conventional geotechnical field testing. Thus, geotechnical engineers can use the information obtained from seismic tests for the analysis of conventional geotechnical soil-structure interaction problems.

1.3 *Serviceability limit state (SLS)*

Geotechnical design is based on the fundamental requirement that a structure and its components are safe under maximum loads and forces. However, the structure must also be capable to serve the designed functions without excessive deformations. The onset of excessive deformations is called serviceability limit state (SLS). SLS is defined as the state beyond which specified service requirements are no longer met. The evolving standard on which geotechnical design in Europe will be based, Eurocode 7 establishes the principles and requirements for safety and serviceability of structures. Traditionally, geotechnical engineers were trained to, and capable of analyzing and designing stability and bearing capacity problems, which require information about the strength of foundation materials. In contrast, designing structures for normal operating conditions (SLS) requires often new, more complex analytical concepts, and more sophisticated soil models. These must account for the variation of soil stiffness (soil modulus) over a wide strain range.

During the recent past, important progress has been made in the development of analytical methods, which can treat even complex loading situation and accommodate sophisticated soil models. The main limitation in the past has been – and in many cases still is – the difficulty to select realistic deformation parameters for soils. Little effort is often spent on verifying that the chosen soil parameters realistically represent the actual foundation conditions, even in the case of important and complex projects. One of the most difficult soil parameters to assess is soil stiffness (modulus), and its variation with stress (strain).

Under undrained conditions, deformations in fine-grained soils occur quickly. However, the assumed soil stiffness has an important effect on the calculated response, i.e. influences the interaction between the construction element (e.g. a pile or sheet pile) and the surrounding soil.

1.4 *Simplified Soil Models*

In the early days of soil mechanics, deformation properties of soils were chosen based on practical experience, i.e. from the observation and back-analysis of actual projects. It was known early on that almost all soils behave “non-linearly” even at low stress levels. However, suitable investigation methods (in the field and laboratory) did not exist. Therefore, empirical correlations were developed between the elastic modulus, E (Young’s modulus) of the soil and soil parameters obtained from various testing methods, such as the Standard Penetration Test (SPT) or the cone penetration test (CPT).

In order to calculate the contact pressure and stress distribution below footings, it was necessary to develop simplified soil models, e.g. by replacing the supporting soil by a bed of equally spaced and equally compressible springs, Terzaghi & Peck (1948). In spite of this crude assumption, the concept has found wide-spread acceptance and is still used by many geotechnical engineers. The ratio between the applied stress and the corresponding settlement is known as the “coefficient of subgrade reaction”, k_s which is defined as

$$k_s = \frac{p}{s} \tag{1}$$

where p, kg/cm^2 = load and s, cm = deformation of the subsoil (subgrade). In an elastic material, the settlement below the center of a rigid plate can be calculated from

$$s_0 = \frac{r\pi p}{2E}\left(1-\nu^2\right) \tag{2}$$

where r = plate radius, E = modulus of elasticity and ν = Poisson’s ratio. With the definition of the coefficient of subgrade reaction according to equation 1, the following relationship between k_s and E can be obtained

$$k_s = \frac{E}{2r\left(1-\nu^2\right)} \tag{3a}$$

Note that this relationship depends on the plate size, which normally is 50 to 70 cm. For fine-grained soils it can be assumed that ν = 0.5, which gives the following expression

$$k_s = \frac{E}{1.5r} \tag{3b}$$

The modulus of subgrade reaction is equivalent to the spring constant, which is commonly used to analyze the dynamic response of foundations on elastic material. The spring constant represents the load required to move the foundation block in the direction of the force, exerted by the load through a distance 1.

In Sweden, it is frequently assumed that $k_s = 80\tau_{fu}$ where τ_{fu} = the undrained shear strength determined by the field vane test and corrected for plasticity. Broms (1963) proposed the following, still widely used, relationship for the calculation of the lateral resistance of piles in clay. The modulus of subgrade reaction, k_0 for a rigid plate with a side length of 1.0 m, and assuming ν = 0.5, can be estimated from

$$k_0 = 1.67E_s \tag{4}$$

where E_s is the equivalent modulus of elasticity. The value of E_s depends on the stress level. At 50 % of the failure load (factor of safety = 2) at short-term loading (undrained conditions), E_s is equal to 50 – 200 times the undrained shear strength, τ_{fu}.

2 ESTIMATION OF SOIL MODULUS

2.1 *Elastic Modulus*

At undrained loading, the elastic modulus, E_u reflects the immediate settlements which occur before consolidation starts. Due to difficulties of determining the deformation characteristics by laboratory tests, empirical relationships are frequently used. It is often assumed that E_u is related to the undrained shear strength. Bjerrum (1972) has proposed that the ratio E_u / τ_{fu} ranges from 500 to 1500, where τ_{fu} is determined by the vane shear test. The lowest value is for highly plastic clays, where the applied load is large. The highest value is for clays of low plasticity, where the added load is relatively small. A wide range of values has been proposed in the literature, cf. Holtz & Kovac (1981).

In Fig. 1, the ratio of the elastic modulus E_f normalized by the undrained shear strength, τ_f is plotted against plasticity index, *PI*. There is much scatter for *PI* below 50 and not much data available for higher *PI* values. The scatter is not surprising, considering the different methods used to measure the undrained shear strength and the stress level, at which the modulus values were determined. The above given range of values and the data shown in Fig. 1 are of little benefit for design.

Figure 2 shows for the case of normally consolidated clays the variation of the normalized modulus E_u/s_u as a function of the applied shear stress, $\tau_n / \mathrm{s_u}$, after Ladd et al. (1977). The elastic modulus decreases with increasing shear stress and this effect can explain to some extent the large scatter of values in Fig. 1. The normalized modulus decreases with increasing plasticity index. At low shear stress level (0.2), the $E_u / \mathrm{s_u}$ ratio varies between 100 – 1500, and decreases at higher shear stress level (0.8) to 25 – 700.

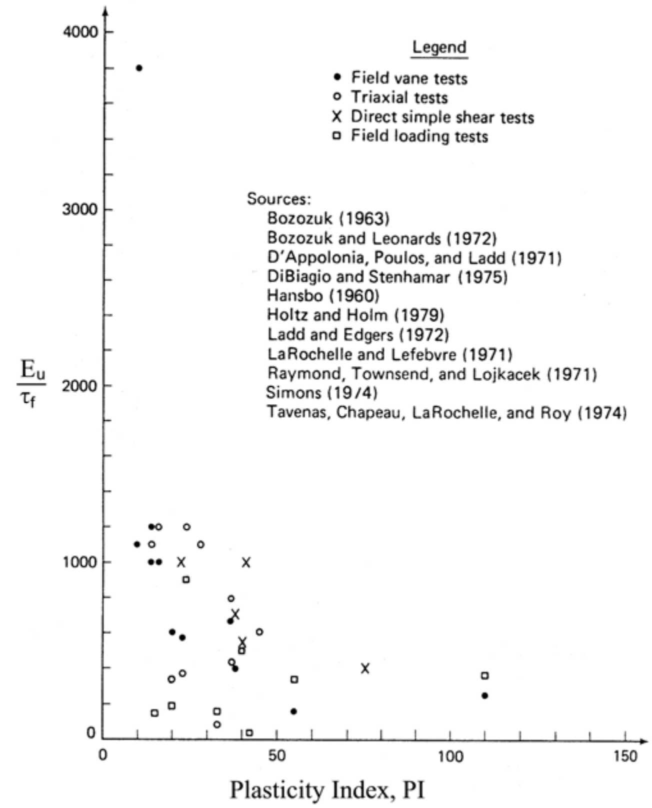


Figure 1. The ratio E_u / τ_{fu} versus plasticity index, PI as reported by several authors, Holtz & Kovacs (1981).

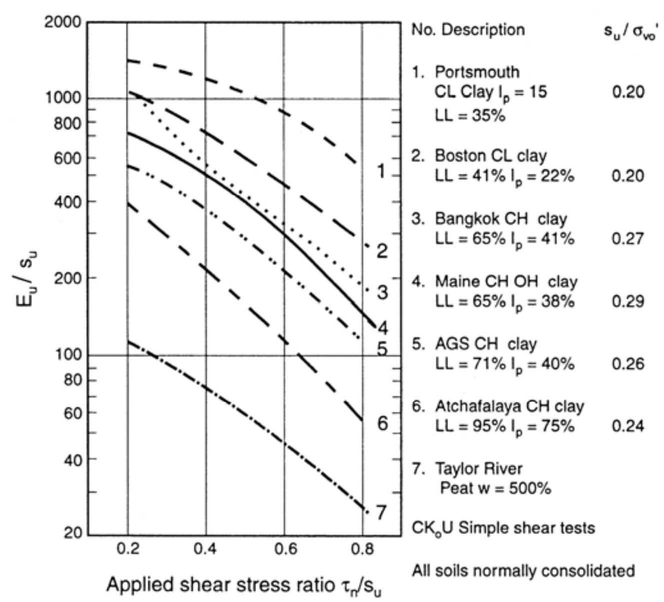


Figure 2. Modulus ratio as a function of the shear stress ratio, from Lunne et al. (1997), after Ladd (1977). Note the semi-logarithmic scale.

2.2 *Definitions*

For the case of an elastic material, Hooks law applies, which defines the relationship between the vertical compression ε_z and the axial stress σ_z

$$\varepsilon_z = \frac{\sigma_z}{E} \tag{5}$$

where E is Young's modulus of elasticity. The ratio between strains in the three directions is given by

$$\varepsilon_x = \varepsilon_y = -\nu\varepsilon_z \tag{6}$$

where ε_x and ε_x are the strains in the directions x and y, respectively and ν is Poisson's ratio. If the shear stress τ_{zx} is applied to an elastic cube, shear distortion γ_{zx} is related to the shear stress according to

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \tag{7}$$

where G is the shear modulus. From equation 5 and 7, the relationship between the modulus of elasticity E and the shear modulus G is obtained

$$G = \frac{E}{2(1+\nu)} \tag{8}$$

The relationship depends thus on Poisson's ratio, ν which needs to be assessed. It is commonly assumed that for undrained conditions in fine-grained soils, $\nu = 0.5$. However, this assumption is not necessarily valid at small strain levels, where ν can be significantly lower (0.15 – 0.3). This aspect can have important consequences when interpreting the results of small-strain tests, but it is usually not recognized when applying equation 8.

2.3 *Definitions of Shear Modulus, G*

The value of the shear modulus depends on the strain level (or the applied shear stress level, i.e. the factor of safety), cf. Fig. 2. In Fig. 3 a typical shear stress-shear strain relationship is shown for fine-grained soils at undrained loading. Three commonly used definitions of the shear modulus are indicated. At very low stress levels (very low strains), the shear modulus is called the maximum shear modulus, G_{max}. With increasing stress level, the shear modulus decreases, cf. Fig. 2. At a stress level corresponding to 50 % of the failure stress the term G_{50} is frequently used, which corresponds to a factor of safety typical for normal operating conditions. At failure, the shear modulus is defined as G_f.

The stress-strain relationship for the case of repeated loading is shown in Fig. 4. The initial loading curve (G_{max}) and the unloading-reloading curves are shown. It is common practice to define the stress-strain relationship of soils by the secant modulus, G_s. Note that at unloading and re-loading, the modulus is often assumed to correspond to the modulus at initial loading, G_{max}.

Frequently, the soil modulus is normalized by the undrained shear strength, cf. Fig. 2. It is implicitly assumed that a linear relationship exists between soil stiffness and soil strength. However, this assumption is incorrect. For normally consolidated, fine-grained soils, a close correlation exists between the ratio τ_f / σ_v' and PI (Bjerrum, 1972)

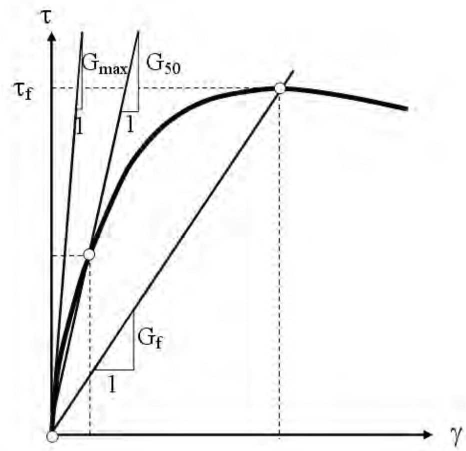


Figure 3. Shear stress – shear strain relationship for fine-grained soil at undrained loading.

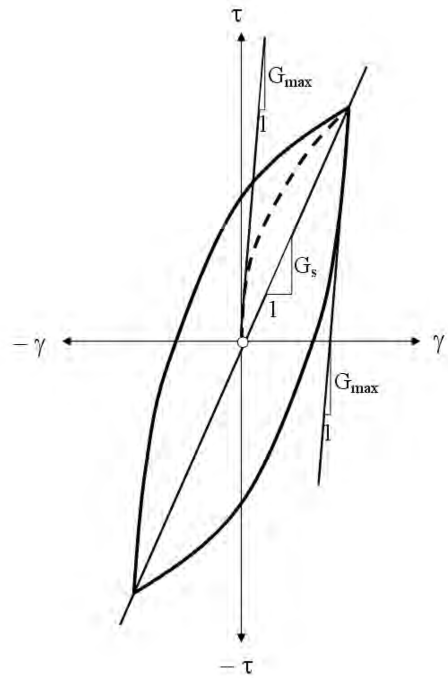


Figure 4. Stress-strain relationship during shear for soils at repeated loading.

$$\frac{\tau_{fu}}{\sigma_v'} = 0.0029PI + 0.13 \tag{9}$$

where σ_v' is the vertical effective stress. On the other hand, the shear modulus at small strains is not related linearly to the effective (overburden) stress, as will be shown in the next section.

3 SHEAR MODULUS AT SMALL STRAINS

3.1 *Empirical Correlations*

Hardin (1978) has proposed the following semi-empirical relationship for the estimation of the shear modulus at small strains, G_{max}.

$$G_{\max} = \frac{625}{0.3+0.7e^2} OCR^k \left(\sigma_0' p_a\right)^{0.5} \tag{10}$$

where e = void ratio, OCR = overconsolidation ratio, k = empirical constant, which depends on PI, $\sigma_0^{'}$ is the mean effective stress and p_a is a reference stress (98.1 kPa). The shear modulus at small strains is thus a function of the square root of the mean effective stress and thus also of the vertical effective stress. Therefore, the assumption of a linear relationship, G/τ_f is not justified.

The mean effective stress $\sigma_0^{'}$ can be determined from

$$\sigma_0^{'} = \frac{(1+2K_0)}{3}\sigma_v^{'} \tag{11}$$

where K_0 is the coefficient of lateral earth pressure at rest (effective stress). An empirical relationship has been proposed for normally consolidated clays, Massarsch (1979)

$$K_0 = 0.0042PI + 0.44 \tag{12}$$

Hardin (1978) has suggested the following relationship for estimating the parameter k from PI

$$k = 0.006PI + 0.045 \tag{13}$$

Equation 10 for estimating G_{max} is in reasonable agreement with measured values for soft clay and silt (Andreasson, 1979, Bodare, 1983, Langö, 1991, Länsivaara, 1999, Larsson & Mulabdic, 1991, Massarsch, 1985, Vucetic & Dobri, 1991).

3.2 *Determination of Shear Modulus at Small Strains*

The shear modulus at small strain can be determined accurately in the field and in the laboratory. In the field, the seismic down-hole and/or cross-hole test have become routine methods, while the SASW-method is becoming increasingly popular. A description of the different seismic methods and their practical application is given by Stokoe & Santamarina (2000). In Scandinavia, the dynamic plate load test has been used by several investigators, (Andreasson, 1979, Bodare, 1983).

Seismic and dynamic laboratory tests have been described in the literature, e.g. Woods and Henke (1981) and Woods (1994). An interesting development is the bender element measuring technique, which can be combined with conventional laboratory testing methods, such as the triaxial and oedometer test, (Dyvik & Olsen, 1989).

The measuring accuracy of conventional laboratory tests has also improved and stress-strain measurements can now be performed at very low strain levels, during triaxial, simple or direct shear tests.

A high-precision torsional shear test was developed at the University of Kentucky, Drnevich & Massarsch (1979). The unique feature of this device at that time was that the shear modulus could be measured with high accuracy at shear strains as low as 0.001%. The strain rate of the torsional shear test was 0.1 Hz, thus more than one order of magnitude lower than that of a resonant column test. Comparative tests on clays, silty sands and sands have shown that at small strains ($\leq 0.001\%$) the modulus is almost independent of frequency and thus of strain rate. This aspect will be discussed below in more detail.

3.3 *Correlation between G_{max} and τ_{fu}*

Döringer (1997) analyzed data from seismic field and laboratory measurements on fine-grained soils. The tests were evaluated, using the concept presented in the previous section. Substituting equation 9 into equation 10, and inserting appropriate values for k and K_0, as well as by replacing void ratio, e by water content w_n (assuming saturated conditions), the relationship given in equation 14 is obtained. Note in this relationship G_{max} is normalized by the square root of the undrained shear strength, τ_{fu}.

$$\frac{G_{max}}{\sqrt{\tau_{fu}p_a}} = \frac{625}{0.3+0.7e^2}OCR^k\sqrt{\frac{(12)\quad 1+2K_0}{3(0.0029PI+0.13)}} \tag{14a}$$

$$\frac{G_{max}}{\sqrt{\tau_{fu}p_a}} = \frac{625}{0.3+0.7\left(w_n\frac{\rho_s}{\rho_w}\right)^2}OCR^k\sqrt{\frac{1+2K_0}{3(0.0029PI+0.13)}} \tag{14b}$$

where w_n = natural water content, ρ_s = density of solid particles and ρ_w = density of water. The normalized shear modulus is shown in Fig. 5 as a function of the water content, for different values of the plasticity index, PI and assuming normally consolidated soil (OCR = 1). It is apparent that the normalized shear modulus decreases markedly when the water content of the soil increases. The reduction of shear modulus is less pronounced at higher water content.

In Fig. 5 are also shown the results of seismic measurements from field and dynamic laboratory tests in a wide variety of soils, reported in the literature, Döringer (1997). In spite of the fact that different methods were used to determine the undrained shear strength, the data follow the semi-empirical relationship. Modulus values from field measurements are generally about 10 to 20% higher than those from laboratory measurements.

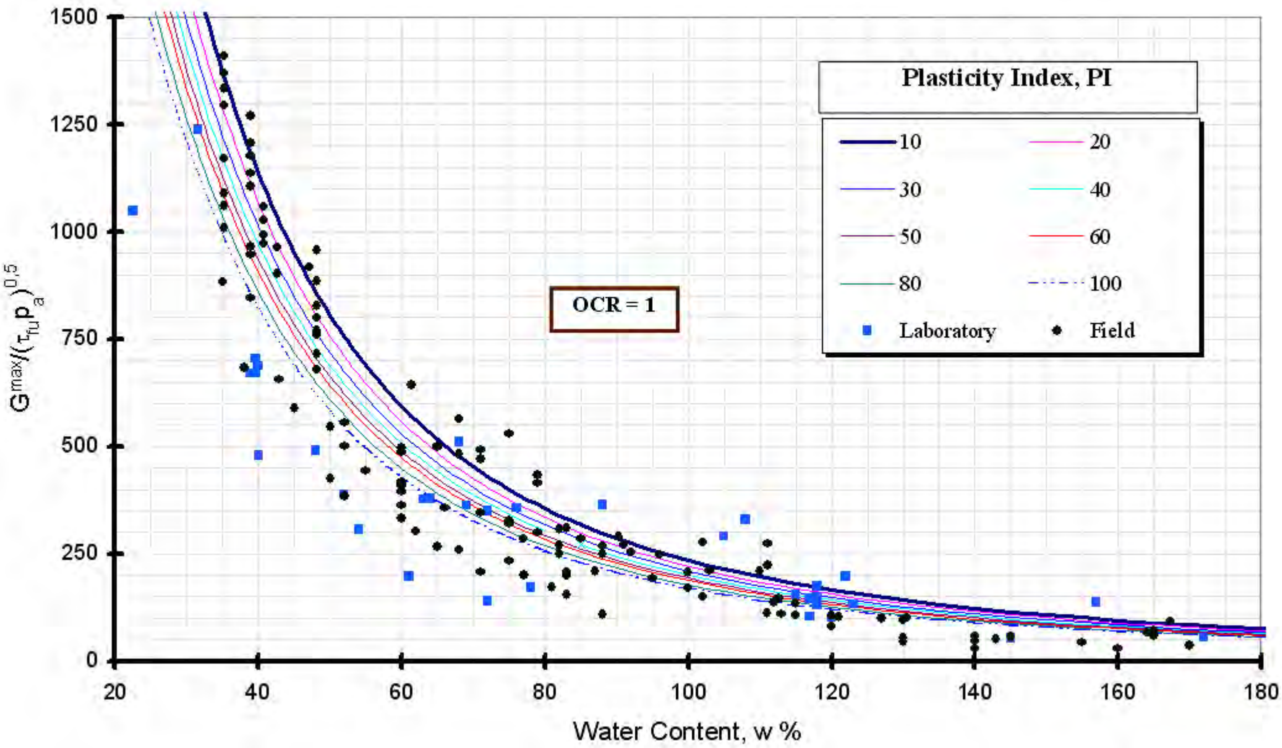


Figure 5. Relationship between the normalized shear modulus at small strains, G_{max} and the water content, cf. equation 14b; from Döringer (1997).

It is apparent that water content (and thus void ratio) has a strong influence on the small-strain modulus. The normalized shear modulus (at small shear strain) is much higher in silty clays and silts than in clays, and can range from 1000 – 2000. In the case of low-plastic clays (w_n = 20 %), the ratio is in excess of 1500 but decreases to 200 when w_n approaches 100 %. The value can be even lower in organic soils. The large scatter of values shown in Fig. 1 and Fig. 2 is thus not surprising.

Equation 14 can be used to estimate the shear modulus and thus also the shear wave velocity. In the case of normally consolidated soft clay with τ_{fu} = 15 kPa, w_n = 80%, and PI = 60, the normalized shear modulus ratio $G_{max}/(\tau_{fu}p_a)^{0.5}$ = 280. In this case, the rigidity index G_{max}/τ_{fu} = 230. Assuming that at small strains ν = 0.3 then E_{max}/τ_{fu} = 600. This value is in reasonable agreement with the range of modulus values at low shear stress level, shown in Fig. 2.

4 EFFECT OF STRAIN ON SHEAR MODULUS

4.1 *General trends*

The shear modulus is affected by stress level and thus by strain level. The measuring accuracy of conventional laboratory testing devices is limited and these can usually not measure G_{max}. On the other hand, the Resonant Column (RC) test can measure shear strain levels down to 10^{-4} % or lower with high precision. Figure 6 shows the results of a RC test on a reconstituted sample of silty clay, Drnevich & Massarsch (1979). The test was performed at a vibration frequency of approximately 30 Hz. At shear strains lower than 10^{-3} %, the shear modulus is almost constant (G_{max} = 77 MPa). However, with increasing shear strains, the modulus decreases markedly and is at 0.1% shear strain 24 MPa, i.e. only 30 % of the maximum value. In conventional laboratory tests, the first data readings would usually be taken at this strain level!

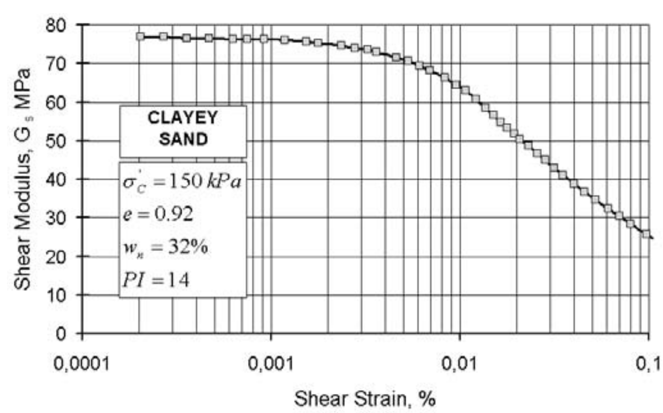


Figure 6. Change of shear modulus with shear strain determined from resonant column test, after Drnevich & Massarsch (1979).

It is thus not surprising that conventional laboratory tests grossly underestimate soil stiffness.

Massarsch (1985) reported results from resonant column tests on a variety of fine-grained soils. Figure 7 shows these results with the normalized shear modulus G_s/G_{max} as a function of shear strain in linear scale. It can be seen that PI has a strong influence on the degradation of the shear modulus. The shear modulus decreases more rapidly in low-plastic soils.

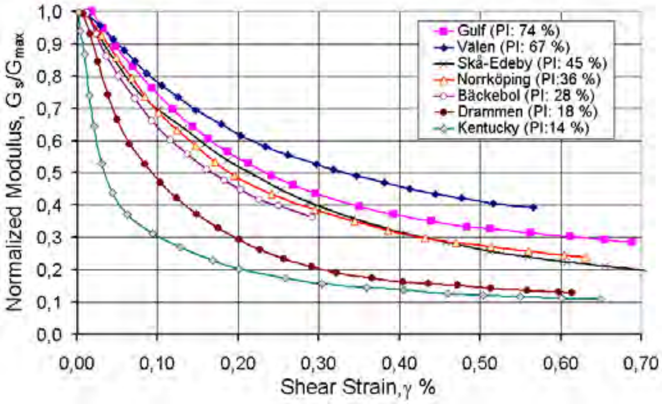


Figure 7. Normalized stress-strain relationship of silts and clays, determined from RC test, (Massarsch, 1985).

4.2 *Stress-Strain Behavior*

The stress-strain behavior of fine-grained soils has been investigated extensively in the areas of soil dynamics and earthquake engineering. Recommendations have been given for estimating the shear modulus as a function of shear strain (Kovacs et al., 1971, Seed & Idriss, 1970). The most widely used correlation between was proposed by Vucetic and Dobry (1991).

Based on a review of laboratory test data published in the literature, they published stress-strain curves, which are shown in Fig. 8. The effect of soil plasticity and number of loading cycles on the stress-strain relationship are also indicated. It can be concluded that in fine-grained soils the plasticity index, *PI* is the most important parameter for the stress-strain behavior and thus the modulus reduction curve. Soils with higher plasticity generally exhibit a more linear stress-strain behavior. The number of strain cycles also affects the soil modulus, which decreases as the number of cycles increases. Since the stress-strain curves are given in chart form this complicates their application in numerical analysis.

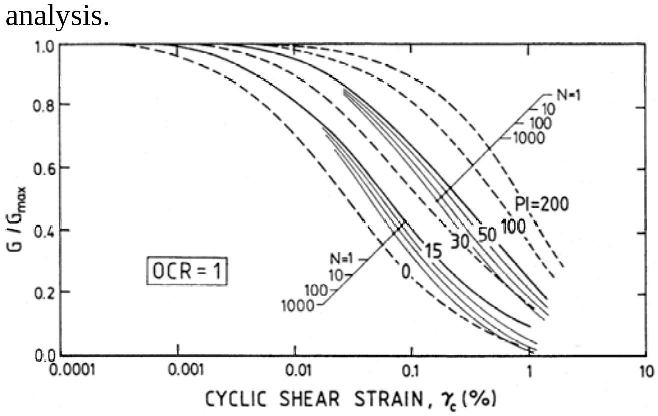


Figure 8. Variation of the normalized shear modulus for normally consolidated soils as a function of the cyclic shear strain, (Vucetic & Dobri, 1991). Indicated is the effect of the plasticity index, *PI* and the number of loading cycles..

Döringer (1997) analyzed stress-strain data published in the literature (mainly RC tests) and performed a regression analysis. A modulus reduction factor, $R_m = G_s/G_{max}$ was used to define the reduction of the shear modulus G_s at three shear strain levels, 0.1, 0.25 and 0. 5 %, cf. Fig. 9.

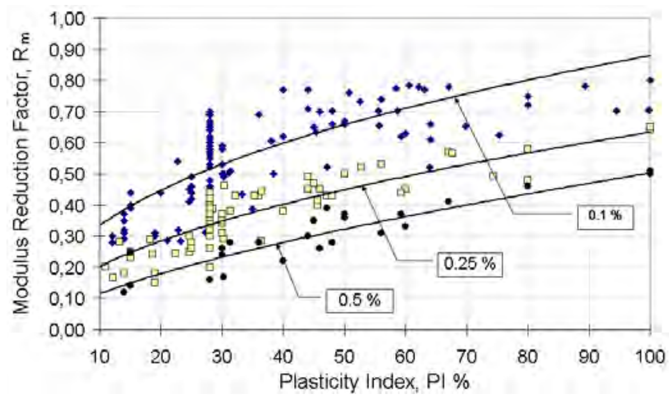


Figure 9. Modulus reduction factor, $R_m = G_s/G_{max}$ as function of the plasticity index, *PI* at three strain levels, (Döringer, 1997).

In spite of the variation in quality of the background material and the wide range of tested soils, a reasonable correlation was obtained. The test results are also in good agreement with those reported by Vucetic & Dobri (1991). The modulus reduction factor R_m decreases rapidly in the case of silty soils. For a soil with $PI = 20$ % at $\gamma = 0.1$ %, the shear modulus is 0.45 G_{max}, and at $\gamma = 0.5$ % the value is 0.15 G_{max}, respectively.

4.3 *Proposed Stress-Strain Model*

Rollins et al. (1998) have compiled stress-strain data for sandy and gravely soils and proposed the following relationship for the variation of the normalized shear modulus G_s/G_{max} with shear strain γ (%)

$$\frac{G_s}{G_{\max}} = \frac{1}{\left[1.2 + 16\gamma\left(1 + 10^{-20\gamma}\right)\right]} \tag{15}$$

The data shown in Fig. 9 for fine-grained soils were analyzed using a modified relationship

$$\frac{G_s}{G_{\max}} = \frac{1}{\left[1 + a\gamma\left(1 + 10^{-\beta\gamma}\right)\right]} \tag{16}$$

where the coefficients α and β were determined empirically from Fig. 9. The correlation between these coefficients and *PI* is shown in Fig. 10.

For clays with $PI = 20 - 40$, a typical range of values for $\alpha = 6.5 - 4.0$, and $\beta = 0.75$ - 0.9, respectively. Equation 16 defines the stress-strain behavior of fine-grained soils numerically, which facilitates its use in analytical models.

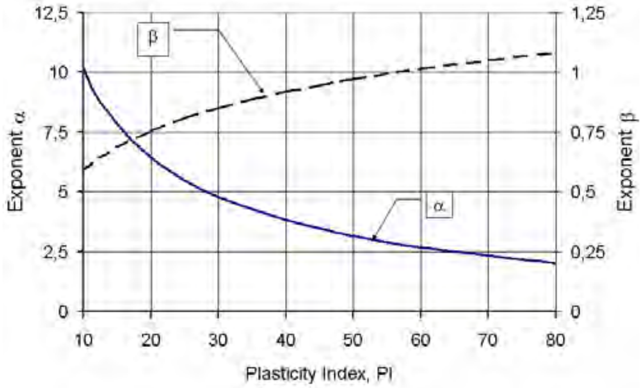


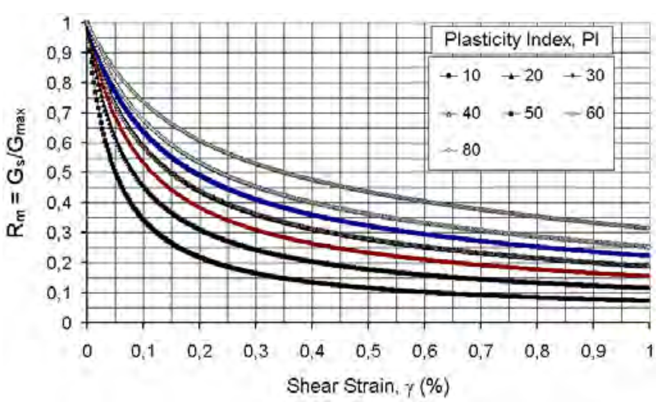
Figure 10. Correlation between PI and parameters α and β, cf. equation 15.

The variation of the normalized shear modulus is shown as a function of shear strain, for different values of *PI*, both in linear as well as in semi-logarithmic scale, cf. Fig. 11 a and b. The data presented in Fig. 11 are in good agreement with previously proposed correlations by Vucetic and Dobry (1991). This is not surprising as the present investigation used part of the same database. However, the present investigation includes additional data, mainly from Scandinavia.

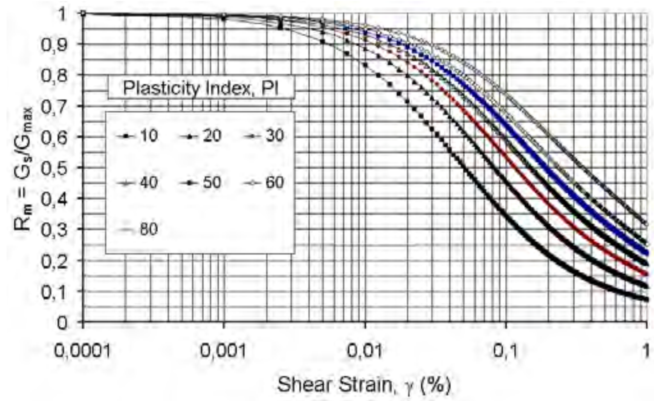
5 EFFECT OF STRAIN RATE ON SOIL STIFFNESS

5.1 *Some Observations Concerning Strain Rates*

One reason why geotechnical engineers have been reluctant to use stress-strain data from seismic investigations and RC tests was their notion of a "seismic modulus", which is only applicable for dynamic problems (i.e. at high strain rates). In the following, it will be shown that this is a misconception, and that in the case of seismic tests at small strains, the loading rate is slow, and in many cases slower than during conventional geotechnical testing.



a) Linear scale

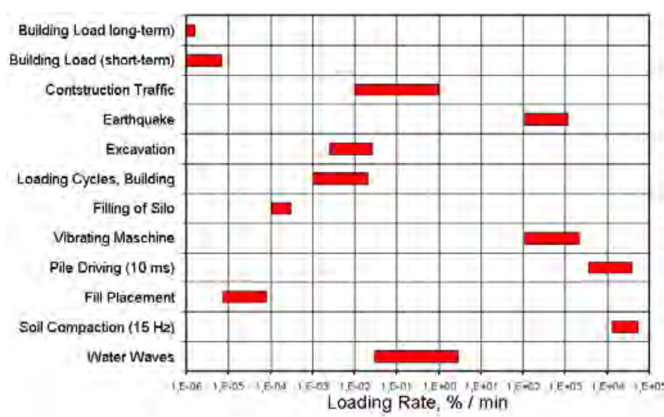


b) Semi-logarithmic scale

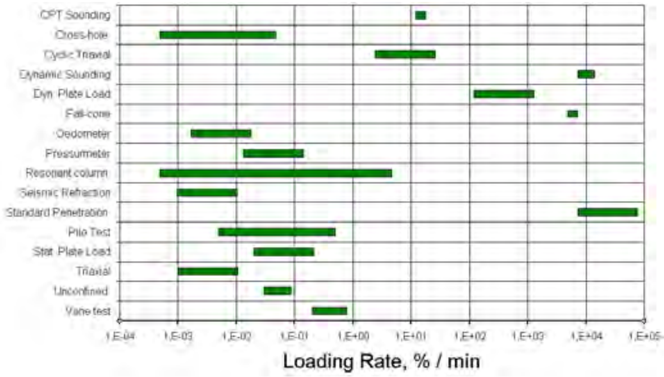
Figure 11. Variation of the normalized shear modulus as a function of shear strain for different values of *PI*, cf. equation 16.

In order to illustrate the problem, typical values of loading rate during different types of construction activities and for commonly used geotechnical investigation methods were estimated, Fig 12 a & b. It should be pointed out that these estimates are not very accurate and merely intended to illustrate the point. In order to assess the loading rate, it has been assumed that deformations (shear strain, %) occurs with time according to a sinusoidal relationship (1/4 of a sine wave). The secant from origin to peak strain was then calculated and this value was used as the average loading rate (either %/min or %/s).

It is noteworthy that the average loading rates stretch across more than 10 orders of magnitude (%/min). Even more interesting is that the loading rate at conventional field and laboratory tests, which are used on a daily basis for "static" design purposes, is significantly higher than that of seismic and cyclic soil tests. For instance, the loading rate of the three most common geotechnical field tests (SPT, CPT and vane test) is significantly higher than that of a seismic cross-hole test.



a. Construction activities



b. Investigation methods

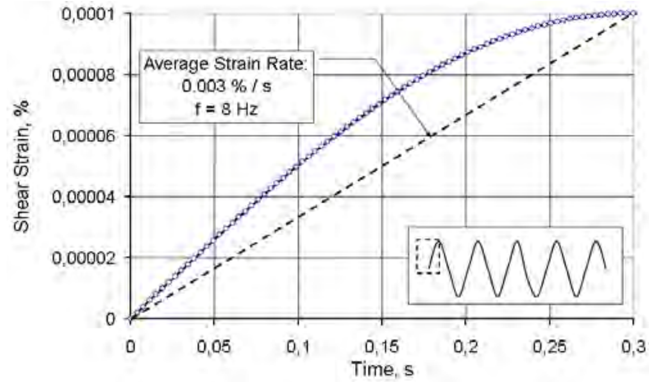
Figure 12. Estimated range of average loading rate during construction activities (a) and geotechnical investigations (b). The loading rate was estimated using a ¼-sine curve.

The loading rate at small strain levels of the laboratory RC test is comparable to that of an undrained shear or triaxial tests, and significantly lower than that of the fall-cone or the laboratory vane test.

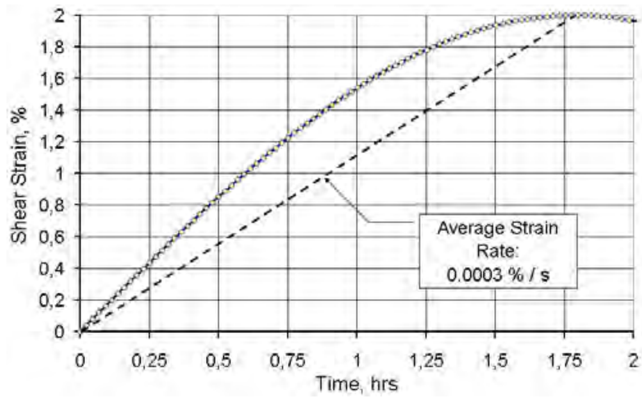
5.2 *Loading Rates during Seismic Testing*

The effect of the rate of loading on the undrained shear strength has been discussed in the literature, e.g. Bjerrum (1972). However, the effect of loading rate on soil stiffness (shear modulus) has not been the focus of much attention. More than 20 years ago, at the ICSMFE in Stockholm in 1981, the following topic was discussed - *The shear modulus determined from "seismic tests" is generally referred to as a "dynamic modulus". Its significance for static geotechnical engineering is not yet generally appreciated. It can be shown, however, that at small shear strain, the "dynamic" shear modulus actually is determined at a strain rate which corresponds to static loading conditions*.- (Massarsch, 1982). After more than 20 years, this fact is still not appreciated, although it has potentially very important consequences. To illustrate this important point, reference is made to Fig. 6, which shows the results of a RC test on silty clay. The vibration frequency was approxmately 30 Hz. At a shear strain level of 0.0002 %, the rate of loading is 0.024%/s. This is comparable to a conventional undrained compression test. At 0.0002 % shear strain, the measured shear modulus was 76 MPa. The threshold value, where the shear modulus started to decrease, was 0.002 %. Since the test was performed at a constant vibration frequency, the rate of loading increased between these two values by one order of magnitude, without any discernable effect on soil stiffness. This fact has been demonstrated by numerous investigations using the RC test. Thus strain rate has little or negligible influence on the shear modulus at low strain level (~ <0.001%).

It is interesting to compare the loading rate of a "dynamic" resonant column (RC) test on soft clay with a "static" direct shear test. In Fig. 13 a & b the rate of loading of the two tests is compared.



a. "Dynamic" Resonant Column test



b. "Static" Direct Shear test

Figure 13. Comparison of average shear strain rate from dynamic RC and static shear test.

Shear strain is plotted against time (seconds and hours, respectively). In the case of the "dynamic" RC test, the average rate of loading at a vibration frequency of 8 Hz and a shear strain level of 0.0001 % is 0.0003 %/s. In the case of a "static" direct shear test, which is performed typically during 1 - 2 hours to failure (2 – 5% shear strain), the average shear strain rate is almost the same (2% during 1.75 hrs: 0.003 %/s). The shear modulus determined at a strain level one order of magnitude lower would remain essentially unchanged if performed at the same strain rate as the laboratory test. Thus the start (at low strain level) and the end of the stress-strain curve (at high strain level) can be established reliably by two tests, which are carried out at the same strain rate.

It is then possible to combine the results of a dynamic and a static test to establish the stress-strain relationship over a large strain range. This concept is illustrated in Fig. 14.

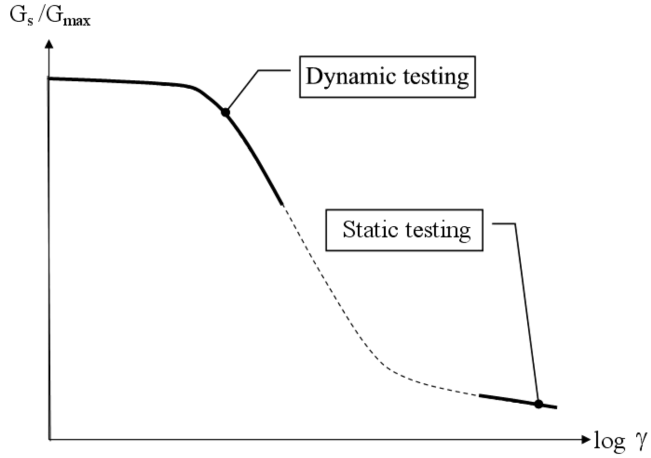


Figure 14. The results of a dynamic test and a static test can be used to establish the stress-strain curve, as they are determined at the same strain rate.

6 CASE HISTORY BÄCKEBOL

6.1 *Geotechnical Conditions*

The Bäckebol site, located north of Gothenburg on the Swedish west coast, has been the subject of numerous detailed geotechnical investigations, (Andreasson, 1979, Fellenius, 1972, Larsson & Mulabdic, 1991, 1991, Massarsch, 1976, Sällfors, 1975, Torstensson, 1973). The depth of the soft, plastic clay exceeds 40 m. Below an approximately 1 m thick dry crust a relatively homogeneous deposit of marine, post-glacial clay is found down to 10 m. The ground water level is located about 1 m below the ground surface and the pore water pressure is hydrostatic below that level. The water content in this layer varies between 70 – 90 % and is slightly above the liquid limit. The plastic limit is around 35 % and the plasticity index around 50 %. At 4 to 5 m depth, the clay is slightly overconsolidated. The coefficient of lateral earth pressure at rest, K_0 has been investigated in the field as well as in the laboratory, (Massarsch & Broms, 1976), cf. equation 12 and at 5 m depth K_0 = 0.6. The density of the clay is 15.5 kN/m^3.

The undrained shear strength has been determined in a comprehensive testing program involving both field vane tests and model pile tests, Torstensson (1973). The sensitivity of the clay at 4 to 5 m depth is around 20. The undrained shear strength at 4 to 5 m depth is 15 kPa, determined from field vane tests and corrected for plasticity. Figure 15 shows the results of field vane tests, which were performed at a depth of 3.75 m at different loading rates. Note that the standard loading rate during a field vane test is normally 1 min to failure.

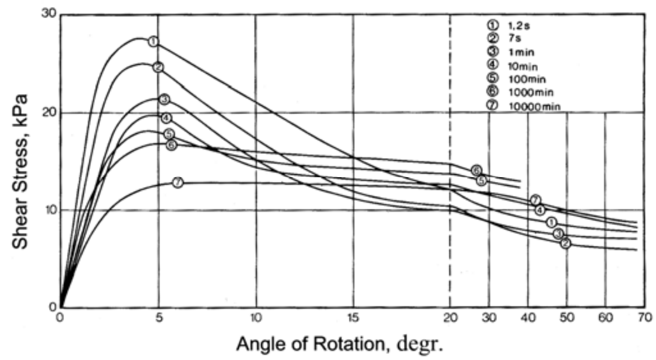


Figure 15. Shear stress from field vane tests as a function of angle of rotation for different loading rates, Torstensson (1973).

If it is assumed that failure occurs at about 1.0 % shear strain during rapid loading, and at about 5 % during slow loading, it is possible to estimate the influence of shear strain rate on the undrained shear strength from Fig. 16. The strain rate during a standard vane test is 0.03 %/s (1.6 %/min). The undrained shear strength in Bäckebol clay increases by about 15 % per log cycle. Thus also the secant shear modulus determined at failure, G_f, will be affected by the rate of loading.

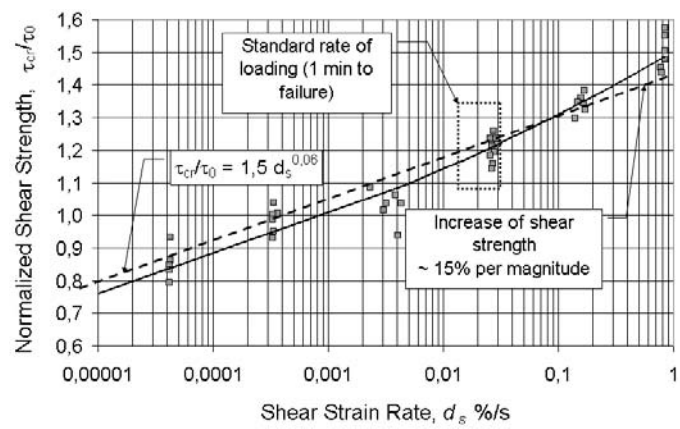


Fig. 16. Influence of shear strain rate on undrained shear strength from field vane tests. The approximate standard loading rate is indicated. Evaluation of results shown in Fig. 16.

Consolidated, undrained triaxial tests were also performed by Torstensson (1973). The undrained shear strength, determined from the deviator stress, is in good agreement with the field vane tests. The stress-strain curve of the tests on a sample from 4.5 m depth was digitized and is shown in Fig. 17.

Failure occurred at 1.25 % axial strain. Shear strain γ is related to axial strain ε_a according to the following relationship, cf. also equation 8

$$\Delta\gamma = (1+\nu)\Delta\varepsilon_a \qquad (17)$$

where ν is Poisson's ratio. Assuming undrained conditions (ν = 0.5) γ = 1.5 ε_a. It is then possible to calculate the equivalent shear modulus, which at failure (1.9 % shear strain) is G_f = 790 kPa.

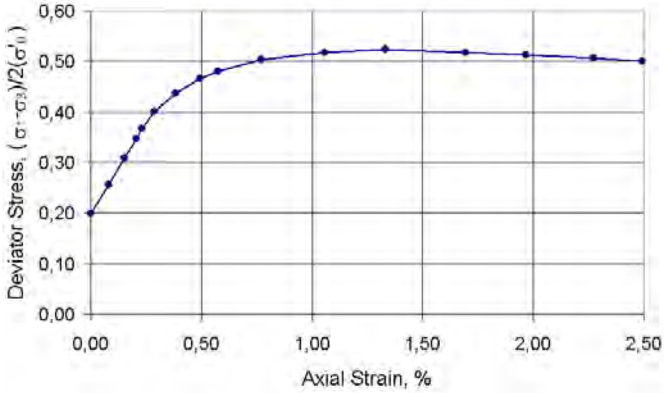


Fig. 17. Stress-strain curve of consolidated, undrained triaxial test from sample at 4.5 m depth, evaluated from test results reported by Torstensson (1973).

6.2 *Seismic Tests*

Different types of seismic and dynamic tests were performed at Bäckebol, (Andreasson, 1979, Bodare, 1983. Larsson & Mulabdic, 1991). These comprised dynamic screw plate tests and different types of seismic DH- and CH-tests. In addition, Andreasson (1979) performed RC tests on solid and hollow soil specimens. Figure 18 shows the results of seismic CH tests with two different types of energy sources. The shear wave velocity at 4 and 5 m depth ranged from 70 to 80 m/s.

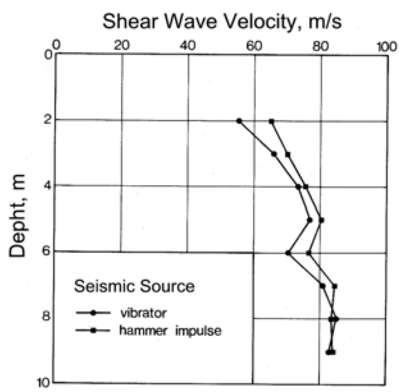
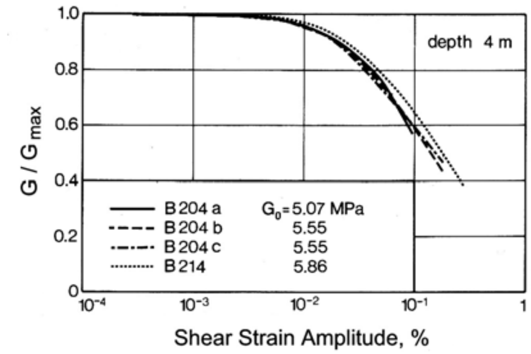
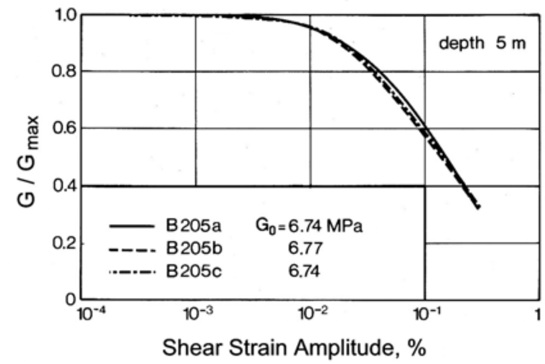


Fig. 18. Results of seismic CH-tests, Andreasson (1979).

Resonant column tests were performed on undisturbed samples from different depths, using a fixed-free device and following standard test procedures, Andreasson (1979). The measuring range of shear strains was 0.0003 – 0.1 %. The undisturbed samples were reconsolidated to the in-situ stress state and thereafter the test was started. Figure 19 a & b show the results of RC tests at 4 and 5 m depth, respectively. The maximum shear modulus was 5.5 and 6.7 MPa at 4 and 5 m depth, respectively. From the RC tests, the modulus reduction curve could be established, Fig. 19.



a. RC-test at 4 m depth



b. RC-test at 5 m depth

Figure 19. Results of resonant column tests, Andreasson (1979).

6.3 *Application of Stress-Strain Concept*

The shear wave velocity determined at small strains from RC tests (65 m/s) was slightly lower than the field values from CH tests (70 - 80 m/s). The normalized stress-strain curves determined at both depths were in good agreement. It is now possible to combine the results from the seismic tests (CH and RC tests), Fig. 18 and 19 with the triaxial tests shown in Fig. 17. Axial strain was converted to shear strain according to equation 17. The data at 4 m depth were used to establish the stress-strain curve according to the concept presented in Fig. 14. The results are shown in Figure 20, which demonstrates that it is possible to determine reliably a stress-strain relationship over a large strain range, from very small strains (0.0001 %) to large (5 %) shear strains. In Fig. 20 is also shown the stress-strain relationship as determined from equation 16, and Fig. 10, assuming PI = 50 (a = 3.17, β = 0.97).

6.4 *Pore Pressure during Pile Driving*

In the vicinity of the Bäckebol test area, comprehensive investigations were performed, aiming to predict the excess pore water pressure during driving of prefabricated concrete piles, Massarsch (1976) and Massarsch & Broms (1981). The prediction model was based on cavity expansion theory and Fig. 21 shows the relationship used in the investigation.

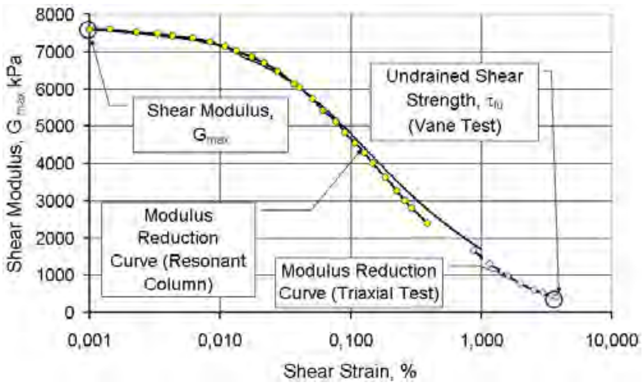


Figure 20. Combination of stress-strain measurements from resonant column and triaxial tests. Fig. 18, 19 and 20.

As is the case in many theoretical prediction models, the results are strongly influenced by the assumed soil parameters. An important in-put parameter is the stiffness ratio (G/τ_f). Considering the uncertainty of soil modulus values as described in the earlier part of this paper (Fig. 1 & 2), it is often difficult to make realistic assumptions on which to base predictions. The stress-strain relationship for Bäckebol, as shown in Fig. 20 suggests that at failure (i.e. close to the pile), the stiffness ratio G/τ_f can be as low as 50 (~790/15), based on triaxial tests. However, at small shear strain level (0.0001%), further away from the pile the ratio increases G/τ_f = 500 (~7600/15). This variation is significant as it corresponds to a factor of 10, which is not negligible even in the case of preliminary studies.

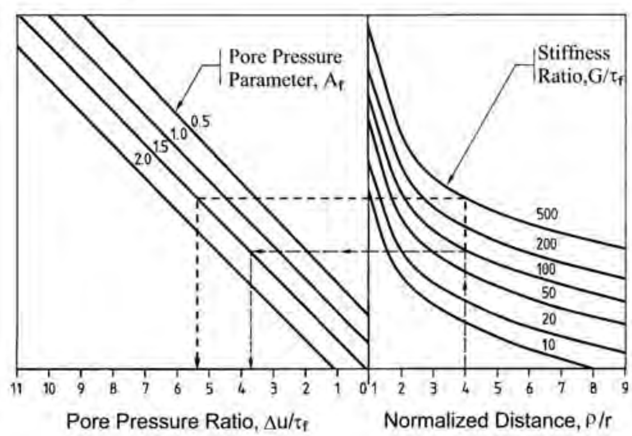


Figure 21. Relationship between the excess pore water pressure in the vicinity of an expanding cavity for different stiffness ratios, Massarsch (1976).

The soil stiffness (G/τ_f) can be established using the stress-strain relationship shown in Fig. 20. The concept presented in this paper made it possible to predict excess pore water pressure more reliably, Massarsch & Broms (1981). The shear strain level at different distances from the pile can be calculated theoretically. In Fig. 21, the range of stiffness ratios (100 – 500) used in the analysis is indicated.

7 CONCLUSIONS

The assessment of deformation properties is an important part of geotechnical design. Simplified concepts, which were developed many decades ago, such as the coefficient of subgrade reaction or spring constants, are still used in practice. These empirically determined values do not take into account fundamental geotechnical concepts, such as the effect of confining pressure or strain level.

Information about the soil modulus published in the literature shows large scatter and is not acceptable for reliable design. In spite of this, many complex projects are analyzed by sophisticated analytical methods, where geotechnical input parameters are chosen based on crude assumptions.

Major progress has been made in earthquake engineering and soil dynamics, and reliable methods exist for the determination of deformation properties of soils even at very low strain levels (down to 0.0001 % shear strain). Based on a comprehensive analysis of published seismic data a surprisingly good correlation was obtained between the shear modulus at small strain and water content (void ratio).

It is well-known that shear strain affects soil stiffness. The reduction of the shear modulus for a wide range of fine-grained soils was determined at three strain levels (0.1, 0.25 and 0.5 % shear strain). The modulus reduction factor is strongly affected by the plasticity index. The shear modulus decreases more rapidly in silts and silty clays, while the effect of shear strain is less pronounced in soils with high plasticity. A numerical relationship is proposed, which makes it possible to establish the stress-strain relationship over a large strain range (from 0.0001 – 0.5 % shear strain).

The soil modulus, determined by seismic or dynamic methods, is often termed the "dynamic" modulus. It is shown that the loading rate during dynamic testing (for instance the resonant column test) at small shear strain levels is slow (0.001 – 0.01 %/sec) and comparable to that of conventional laboratory tests (triaxial and shear tests). Thus, the starting point and the end-point of a stress-strain curve can be established by a seismic or dynamic test (preferably performed in the field) and a static shear test, respectively. The modulus reduction curve can be determined accurately by the resonant column test. In the absence of such test data, the relationship proposed in this paper can be used.

The application of the proposed concept was exemplified using data from a well-documented test site in Bäckebol, Sweden. Good agreement was obtained between measured and predicted soil deformation data.

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