

# Use of CPT for design, monitoring, and performance verification of compaction projects

K. Rainer Massarsch

Geo Risk & Vibration AB, Bromma, Sweden, rainer.massarsch@georisk.se

B. H. Fellenius

Sidney, BC, Canada. bengt@fellenius.net

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ABSTRACT: Soil compaction projects can potentially offer significant savings in terms of cost and time. However, they require the active participation of the geotechnical engineer throughout the project. An important requirement of many compaction projects is to control total and differential settlement. The definition of project-specific compaction criteria is a key task, the importance of which is not often recognized. A reliable and transparent method of calculating settlement in granular soils is the tangent modulus method. By assessing soil compressibility prior to the compaction work it is possible to determine whether, and to which degree, soil compaction will be required. A method is presented by which the modulus number can be determined from CPT results adjusted for mean confining stress. The compactability of soils can be assessed, based on the cone stress and sleeve friction records. An important aspect of soil compaction is the increase in horizontal stress, which results from the compaction-induced overconsolidation of the soil. A method is described which makes it possible to take this preconsolidation effect into account. The proposed concepts are illustrated by examples.

#### 1 INTRODUCTION

Natural or man-made ground, consisting primarily of granular material (silt and sand), can be compacted by dynamic and/or cyclic methods. Also static preloading can be used, but this method is not suitable on projects where soil deposits are subjected to dynamic and cyclic loading. Designing a soil compaction project is different to designing a conventional foundation project, such as a piled foundation. In order for the design to be successful, it requires the active participation of the design engineer during all phases of the project, from planning and field trials through the execution of the compaction work. An important aspect is the close cooperation between the geotechnical designer and the foundation contractor.

Planning and implementing of a soil compaction project is an ongoing process, comprising the following steps: a) perform a geotechnical analysis considering the effect of static and/or dynamic/cyclic loading on the unimproved ground; b) if the project requirements with respect to stability and/or settlement are not met, determine the extent of compaction necessary to meet the foundation requirements, c) select potentially suitable compaction method(s) based on experience and/or field trials, and d) verify that the specified compaction criteria have been achieved.

This paper describes how the cone penetration test (CPT) can be used during the above described phases of a soil compaction project.

#### 2 DEFINITION OF COMPACTION CRITERIA

An important decision for the planning and implementation of a compaction project is to specify compaction criteria. Unfortunately, even on large projects, compaction specifications are often based on empirical rules or texts lifted from previous projects specifications. For instance, the required degree of compaction is frequently expressed in terms of density index,  $I_D$ , (earlier on referred to as, "relative density",  $D_r$ , which term is still used, but now usually assumed to refer to compactness condition determined from in-situ tests, such as the SPT or CPT). The definition of the density index,  $I_D = (e_{\text{max}} - e)/(e_{\text{max}} - e_{\text{min}})$  is based on the assumption that the void ratio of the soil can be reliably determined for the "maximum" and "minimum" density of a natural soil. Over the years, "relative density" has been used as a parameter to describe geotechnical parameters of sand deposits and correlations have been developed to estimate the angle of internal friction, liquefaction potential, and soil modulus. However, as has been shown by many, e.g., Tavenas and LaRochelle (1972), it is an ambiguous and qualitative procedure, which, whenever possible, should be avoided. This fact is recognized in the Eurocode EN 1997-2, Part 2, which does not define density index in terms of numerical values.

Considering the progress in geotechnical testing and analysis during the past decades, it is unnecessary to base the design on qualitative formulations of soil parameters. In granular soils, such as silt, sand, and gravel, penetration testing provides, when properly applied, the most reliable basis for planning, execution, and supervision of soil compaction projects. This paper describes how the cone penetration test with pore water pressure measurement (CPTU) can be used during all phases of a soil compaction project.

On most soil compaction projects, the critical design consideration is total and differential settlement. However, settlement analyses are often still based on over-simplified empirical methods that often are quite unreliable. Uncertainties associated with settlement analyses depend usually not on the choice of numerical method as much as on the selection of appropriate soil parameters (deformation modulus values) to be used in the analysis. This paper describes how settlements can be estimated based on the tangent modulus method, which is a transparent and generally accepted concept. A method is presented on how the deformation parameters for settlement analysis can be determined from CPTU results. The advantage of this approach is that the design can be based on quantitative information, which makes it possible to assess settlements prior to and after compaction. Thus, compaction criteria can be based on CPTU results rather than ambiguously described soil properties, such as the density index.

## 3 SETTLEMENT ANALYSIS ACCORDING TO TANGENT MODULUS METHOD

The tangent (1-D) modulus method was introduced by Janbu (1963) and is described in the geotechnical literature, CFEM (1992) and Fellenius (2012). This unified method of settlement calculations is based on the tangent modulus,  $M_t$ , defined by

$$M_{t} = \frac{d\sigma}{d\varepsilon} = m \,\sigma_{r} \left(\frac{\sigma_{v}'}{\sigma_{r}}\right)^{(1-j)} \tag{1}$$

where  $d\sigma$  = change of stress,  $d\varepsilon$  = change of strain; m = Janbu modulus number (dimensionless);  $\sigma_r$  = a reference stress (equal to 100 kPa);  $\sigma_v$  = vertical effective stress; j = stress exponent. The modulus number has a direct mathematical relation to the conventional  $C_c$  and  $e_0$  approach in clay soils (where the stress exponent is zero) and in gravel and till (where linear elastic conditions are assumed and the stress exponent is unity). In soils suitable for compaction, i.e., silty and sandy soils, the stress exponent, j, is approximately equal to 0.5, and Eq. 1 becomes

$$\varepsilon = \left(\frac{2}{m}\right) \left[ \sqrt{\frac{\sigma_1'}{\sigma_r}} - \sqrt{\frac{\sigma_0'}{\sigma_r}} \right] \tag{2}$$

where  $\varepsilon$  = strain; m = Janbu modulus number (dimensionless);  $\sigma_r$  = a reference stress (equal to 100 kPa);  $\sigma_1$  = final vertical effective stress (kPa). The stresses and stress exponent numbers can be readily assessed. The only parameter in Eq. 2, which requires advanced geotechnical knowledge, is selection of the modulus number, m. For fine-grained soils, the modulus number can be determined from oedometer tests on undisturbed soil samples. For granular soils, however, the modulus number must be determined based on judgment. The Canadian Foundation Engineering Manual (CFEM 1992) gives a range of modulus values for preliminary estimates, Table 1.

Table 1. Typical stress exponent and modulus numbers, from CFEM (1992).

Soil Type	Stress exponent, j	Modulus number, m
Till, very dense to dense	1	1,000 - 300
Gravel	1	400 - 40
Sand		
dense		400 – 250
compact	0.5	250 – 150
loose		150 - 100
Silt		
dense		200 - 80
compact	0.5	80 - 60
loose		60 - 40
Clays		
silty clay; hard, stiff		60 - 20
silty clay; stiff, firm	0	20 – 10
clayey silt; soft		10 – 5
soft marine clays and organic clays		20 - 5
Peat	0	5 - 1

It is desirable to correlate the modulus number to in-situ tests, such as the CPT. The cone stress,  $q_T$ , is affected by the effective confining stress and it is necessary to consider this effect when interpreting CPT results. For the depth adjustment of the cone stress, Massarsch (1994) proposed to apply a dimensionless adjustment factor,  $C_M$ , to the cone stress according to Eq. 3, based on the mean effective stress  $\sigma'_m$ 

$$C_M = \left[\frac{\sigma_r}{\sigma_m'}\right]^{0.5} \tag{3}$$

where  $C_{\rm M}$  = stress adjustment factor  $\leq$  2.5;  $\sigma_{\rm r}$  = reference stress = 100 KPa;  $\sigma'_{\rm m}$  = mean effective stress. The mean effective stress is determined according to

$$\sigma_m' = \frac{\sigma_v' \left(1 + 2 K_0\right)}{3} \tag{4}$$

where  $\sigma'_{\rm m}$  = mean effective stress;  $\sigma'_{\rm v}$  = vertical effective stress;  $K_0$  = coefficient of horizontal earth stress. Near the ground surface, values per Eq. 3 increase disproportionally and it is necessary to limit the adjustment factor to a value of 2.5. The stress-adjusted cone penetration stress is expressed in Eqs. 5a and 5b

$$q_{tM} = q_t C_M \tag{5a}$$

$$q_{tM} = q_t \left(\frac{\sigma_r}{\sigma_m'}\right)^{0.5} \tag{5b}$$

where  $q_t$  = unadjusted cone stress (as-measured);  $\sigma_r$  = a reference stress = 100 kPa;  $q_{tM}$  = stress-adjusted cone stress. Based on information from soil compaction projects, Massarsch (1994) proposed the semi-empirical relationship shown in Eq. 6 between the modulus number, m, and the cone stress adjusted for effective confining stress,  $q_{tM}$ .

$$m = a \sqrt{\frac{q_{iM}}{\sigma_r}} \tag{6}$$

where m = modulus number; a = an empirical modulus modifier, which depends on soil type;  $q_{tM} =$  stress-adjusted cone stress;  $\sigma_r =$  reference stress = 100 kPa. The modulus modifier, a, has been determined from the evaluation of extensive field and laboratory data (Massarsch and Fellenius 2002) and shown to vary within a relatively narrow range for each soil type. Massarsch and Fellenius (2002) proposed the values listed in Table 2.

Table 2. Modulus factor, a, for different soil types, Massarsch and Fellenius (2002).

Soil Type	Modulus Modifier, a	
Silt, organic soft	7	
Silt, loose	12	
Silt, compact	15	
Silt, dense	20	
Sand, silty loose	20	
Sand, loose	22	
Sand, compact	28	
Sand, dense	35	
Gravel, loose	35	
Gravel, dense	45	

If the geotechnical analysis shows that the specified settlement criteria cannot be met for the case of unimproved ground, soil improvement by soil compaction is a potentially suitable foundation alternative. In such a case, it is necessary to perform a settlement analysis, determining what cone penetration resistance values (and modulus number) that would result in a condition improved to acceptable settlement. An important advantage of determining the modulus number from CPT data and Eq. 6 is the fact that CPT data are normally available for compaction projects. Thus, by using the above-proposed concept and the CPT records, it is possible to predict settlements of a foundation, that is, to assess the need for compaction. After repeating the cone soundings on completion of the compaction work, the same procedure is applied to verify the success of the work.

#### 4 COMPACTABILITY OF SOILS

The next step of the design process is to determine, which method is suitable for the geotechnical conditions at the site. An important aspect of soil compaction is to achieve homogeneous foundation conditions, thereby reducing the risk of differential settlement, which is equally important as the maximum degree of soil compaction.

Massarsch (1991) proposed that the compactability of soils by vibratory methods can be classified as "compactable", "marginally compactable", and "not compactable" in a CPT chart as indicated in Fig. 3. (It should be noted that the diagram assumes homogeneous soil conditions. Layers of silt and clay can inhibit the dissipation of excess pore pressures and, therefore, reduce the compaction effectiveness).

Figure 1 shows the same compaction boundaries in two CPT charts. Figure 1a presents the cone stress,  $q_t$ , as a function the friction ratio in a conventional soil classification chart with the friction ratio along the abscissa and the cone stress  $(q_t)$  along the ordinate. Because the ranges of cone stress and sleeve friction applicable to compaction projects are relatively narrow, the usual logarithmic scale compression of the abscissa can be dispensed with (Massarsch 1991). Figure 1b presents the same boundaries plotted in an Eslami-Fellenius classification chart which uses the "effective" cone stress, i.e.,  $q_E = q_t - U_2$  plotted against the sleeve friction  $f_s$ . (Eslami and Fellenius 1995; 1997; and Fellenius and Eslami 2000).

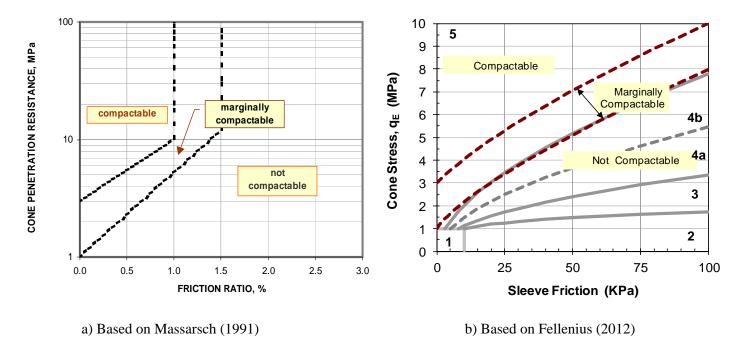


Figure 1. Soil classification for deep vibratory compaction with boundaries for compactable, marginally compactable, and not compactable soils.

#### 5 EXAMPLE OF MODULUS EVALUATION

The following example shows the application of the above method of determining the modulus number. CPT readings are taken intermittently at closely spaced distances, normally every 20 mm, preferably every 10 mm. Figure 2 shows CPTU results in a soil deposit consisting of soft clay overlying silt and sand (Fellenius 2012). For compaction evaluation, it is often beneficial to filter the cone stress values,  $q_t$ , so that any disingenuous peaks and

troughs in the data are removed. The most useful filtering is obtained by a geometric average, usually running over about 0.5 m length.

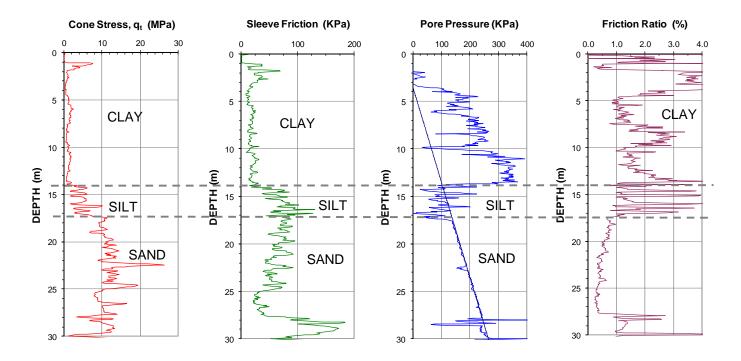


Figure 2. Results of CPTU in soil deposit consisting of soft clay on silt and sand, from a site near Vancouver, BC, Canada, (Fellenius (2012).

The effect of filtering and depth-adjusting the  $q_t$ -values and calculation of the modulus number profile is illustrated in Figure 3, using the CPT sounding of Figure 2. Figure 3 shows the unfiltered and depth-adjusted  $q_t$ -profile, as well as the filtered  $q_t$ -profile determined according to Eqs. 5a and 5b, respectively. The second diagram shows the modulus number distribution calculated according to Eq. 6. The third diagram shows the modifiers ("a") used in the calculation. The modifier for the clay was selected to result in modulus numbers commensurate with those found at an adjacent site where oedometer tests were carried out on recovered clay samples.

### 6 EFFECTS OF SOIL COMPACTION

Soil compaction has the objective of reducing the compressibility of granular soil. This is reflected by the increase in cone stress, based on which the soil stiffness (modulus) can be determined based on the above proposed concept. However, in many compaction projects, almost independently of the used compaction method, it is has been found that the sleeve friction increases following compaction, which fact is an equally important, but often neglected effect of the compaction. In a well-documented case history, Massarsch and Fellenius (2002) showed the increase in cone stress and sleeve friction in a sand deposit before and after compaction, Fig. 4.

The increase in sleeve friction as a result of compaction must be caused by an increase in horizontal stress. The initial horizontal effective stress of normally consolidated soil deposits (e.g., hydraulic fill) can be estimated based on

$$K_0 = 1 - \sin \phi' \tag{7}$$

where  $\phi$  is the estimated effective friction angle.

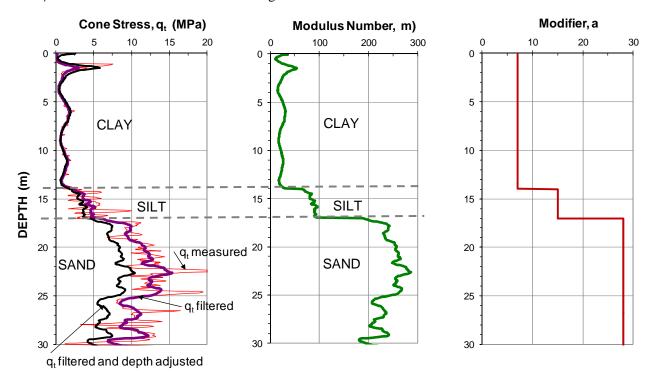


Figure 3. Example of determination of modulus number, m fitted by means of modulus modifier, a for the CPT sounding shown in Fig. 2.

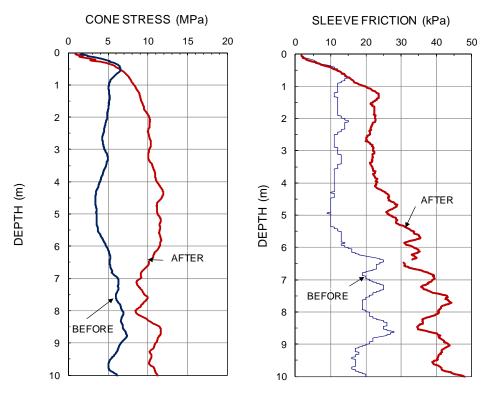


Figure 4. Example of filtered average values of cone stress and sleeve friction from before and after compaction (for two and seven days combined). After Massarsch and Fellenius (2002).

The effective friction angle for normally consolidated sand and silt ranges between 30° and 36°, which according to Eq. 7 corresponds to a  $K_0$ -value of about 0.4 through 0.6. As will be discussed below, soil compaction results in an increase of horizontal effective stress, which is reflected by the increase in the sleeve friction. This effect shall be taken into account when calculating the mean effective stress in Fig. 5b. However, the stress adjusted cone stress,  $q_{tM}$  is less affected by an increase in horizontal stress as the stress adjustment uses the square root of the mean effective stress.

The modulus number, m can be calculated according to the above described procedure, prior to and after soil compaction. Using the data shown in Fig. 4, the variation of the modulus number, m as a function of depth is shown in Fig. 5. Note that due to the stress adjustment of cone stress, the modulus number decreases slightly with depth, in spite of an increase in cone stress. The determined values of the modulus number, m are in good agreement with the range of values for sand, given in Table 3.

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Figure 5. Modulus number, m determined from cone stress in Fig. 4, before and after soil compaction. After Massarsch and Fellenius (2002).

Vibratory compaction results in an increase of the earth stress (often called earth pressure, but should be called stress) coefficient at rest,  $K_0$ . However, in overconsolidated soils, that is, compacted soils, it is more difficult to estimate  $K_0$ . Several investigators have proposed empirical relationships between the earth stress coefficient of normally and overconsolidated sands and the overconsolidation ratio, OCR, as given in Eqs. 8a and 8b

$$\frac{K_1}{K_0} = OCR^{\beta} \tag{8a}$$

$$OCR = \left\lceil \frac{K_1}{K_0} \right\rceil^{\frac{1}{\beta}} \tag{8b}$$

where  $K_0$  = coefficient of earth stress at rest for normally consolidated sand;  $K_1$  = coefficient of earth stress at rest for overconsolidated sand,  $\beta$  is an empirically determined exponent. Based on compression chamber tests, Schmertmann (1985) recommended a value of 0.42 for the  $\beta$ -exponent, and Lunne and Christophersen (1983) suggested 0.45. The Brooker and Ireland (1965) data for sand (soil with low  $I_P$ -value) and the relationships suggested by other authors have been redrawn in Fig. 6, showing OCR as a function of the ratio of earth stress at

rest for overconsolidated and normally consolidated sand,  $K_1/K_0$ , respectively. The diagram shows that a doubling of the earth stress coefficient, which is a relatively small increase, results in an increase of OCR to values ranging from 4 through 7, corresponding to  $\beta$ -exponents ranging from 0.36 through 0.50.

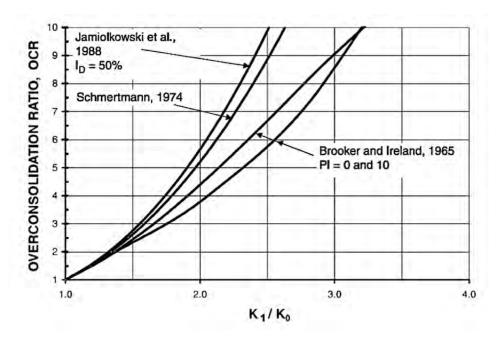


Figure 6. Relationship between  $K_1/K_0$  and OCR for sand. After Massarsch and Fellenius (2002).

The sleeve friction can be approximated from

$$f_s = K_0 \ \sigma_v \ \tan \phi' \tag{9}$$

where:  $f_s$  = sleeve friction;  $\sigma'_v$  = effective vertical stress;  $K_0$  = earth stress coefficient;  $\phi'$  = the effective friction angle for the soil/CPT sleeve interface. It should be noted that the earth stress coefficient adjacent to the penetrating cone can be affected by changes of the stress field and that Eq. 9 should not be used to calculate the earth stress coefficient. The ratio between the sleeve friction after and before compaction,  $f_{s1}/f_{s0}$  can be calculated from

$$\frac{f_{s1}}{f_{s0}} = \frac{K_{01} \, \sigma'_{v1} \, \tan(\phi'_1)}{K_{00} \, \sigma'_{v0} \, \tan(\phi'_0)} \tag{10}$$

where:  $f_{s0}$  = sleeve friction before compaction;  $f_{s1}$  = sleeve friction after compaction;  $K_{00}$  = coefficient of earth stress before compaction (effective stress);  $K_{01}$  = coefficient of earth stress after compaction (effective stress);  $\sigma'_{v0}$  = vertical effective stress before compaction;  $\sigma'_{v1}$  = vertical effective stress after compaction;  $\phi_0$  = friction angle before compaction;  $\phi_1$  = friction angle after compaction. Equation 10 assumes that stress changes caused by the penetrating cone adjacent to the sleeve are the same before and after compaction. The effective vertical stress,  $\sigma'_{v}$  is

unchanged by the compaction process (increase of soil density but reduced layer thickness). The ratio of the earth stress after and before compaction,  $K_{01}/K_{00}$ , can then be estimated from the relationship

$$\frac{K_{01}}{K_{00}} = \frac{f_{s1} \tan(\phi'_0)}{f_{s0} \tan(\phi'_1)} \tag{11}$$

Equation 11 assumes that the earth stress coefficient is directly affected by the change of the sleeve friction and of the friction angle of the soil. To illustrate the importance of the relationship, Eq. 11 is represented in Fig. 7 for sand for which the compaction resulted in a friction angle of 36°, improved from values ranging from 21° through 30° before compaction. The sand is assumed to be normally consolidated before compaction with an earth stress coefficient,  $K_{00}$ , which, according to Eq. 7, is approximately equal to 0.5. The CPT measurements provide the sleeve friction values. As indicated in Fig. 7, the ratio of earth stress coefficients depends primarily on the ratio of sleeve friction and less on the increase in friction angle.

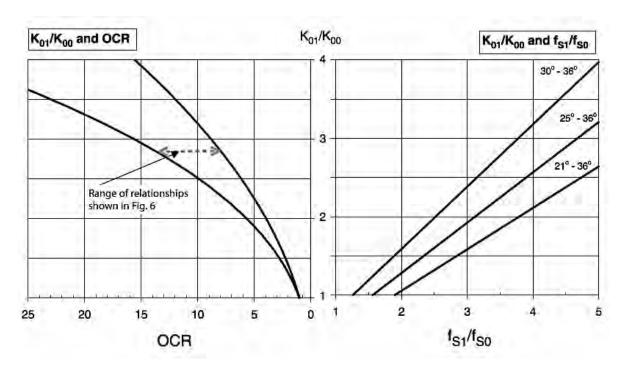


Figure 7. Ratio between sleeve friction before and after compaction to OCR for three levels of increase of the effective friction angle: 21° to 36°, 25° to 36°, and 30° to 36°. After Massarsch and Fellenius (2002).

In the example in Fig. 4,  $f_{s1}/f_{s0}$  is approximately 2, hence  $K_{01}/K_{00}$  varies from 1.1 to 1.6; i.e. assuming  $K_{00} = 0.5$ ,  $K_{01} = 0.55$  to 0.80. As the modulus number is determined from the stress adjusted cone resistance, which depends on the square root of the mean effective stress, the effect of lateral stress change has reduced effect. However, the increase in lateral earth stress causes a preconsolidation effect, which needs to be considered in a settlement analysis.

#### 7 CONCLUSIONS

Soil compaction is potentially a cost-effective and technically flexible solution for many foundation projects on granular soils, where total and differential settlements often are the key design considerations. However, the planning and execution of soil compaction projects requires the active involvement of the geotechnical engineer during all phases of the project, as only then it is possible to take advantage of the possibility to optimize the extent and degree of soil compaction.

Settlement of foundations on granular soils can be analyzed by a variety of methods. A widely used method is based on the tangent (1-D) modulus method. In all settlement analyses, the soil modulus is the key parameter and, in the case of the tangent modulus method, the modulus number. In granular soils, it is usually not possible to obtain undisturbed samples for laboratory testing to establish the soil compressibility for use in settlement analyses. However, with the method proposed in this paper, it is possible to estimate the modulus number based on stress-adjusted cone stress measurements.

An important, but often neglected, consequence of soil compaction is the increase in horizontal effective stress. Sleeve friction measurements after soil compaction are usually several times higher than prior to compaction. This increase in horizontal effective stress demonstrates that the overconsolidation effect caused by the compaction can be quantified by the ratio of the sleeve friction after versus before compaction. The paper demonstrates how the soil modulus and the change in horizontal stress and, therefore overconsolidation, can be quantified based on cone penetration tests. A similar approach should be possible when evaluating the liquefaction susceptibility of loose sands and silts.

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