Determination of Shear Modulus of Soil from Static and Seismic Penetration Testing

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ABSTRACT: The design of foundations on granular soils is usually governed by deformations. Empirical rules are often crude. Two methods for determining deformation parameters of granular soils are described. The first method is based on results from the seismic cone penetration test. The small-strain shear modulus can be measured, from which the modulus at working loads (approximately 0.5 % shear strain) can be estimated. Based on resonant column tests, a modulus reduction factor is determined. The second method is based on the static cone penetration resistance. The tangent modulus method is used to describe soil compressibility. The modulus number can be estimated from the stress-adjusted cone penetration resistance.

1. INTRODUCTION

The design of foundations on soils is usually governed by settlement requirements. Only in exceptional cases, such as when loose, water-saturated soil deposits are affected by dynamic and/or cyclic forces, will strength properties be critical. According to the Eurocode, the design with respect to deformations of geotechnical structure shall be verified for the Service Limit States (SLS). Eurocode EN 1997-1 (CEN 2004) allows checking SLS by two approaches: (a) by calculating the design values of the effect of the actions A_d (e.g. deformation, differential settlement etc.) and comparing them with limiting values L_d by applying the relationship

$$A_d \leq L_d \tag{1}$$

(b) by an alternative method based on comparable experience, showing that a sufficiently low fraction of the ground strength is mobilized to keep the deformations within acceptable limits. Analytical, semi-empirical and numerical methods may be used to verify Equation (1) and to calculate the fraction of the ground strength that is mobilized in the service state. When using Equation (1), the limiting values of deformations should be specified as a design requirement for the supported structures subjected to ground displacements. The assessment of ground deformations requires the knowledge of geotechnical parameters, which control vertical and lateral movements. The most important parameters for the determination of settlement caused by static loading are (a) initial total and effective stresses, (b) stress history prior to loading (preconsolidation stress), (c) changes in effective stress due to loading and (d) deformation parameter (modulus) of the soil. Although the deformation properties of granular soils are important for the analysis of many foundation problems, little guidance can be found in the geotechnical literature. This paper focuses on the determination of deformation parameters of soils. Two different methods will be presented, based on results from seismic and static cone penetration tests.

A settlement analysis requires knowledge of soil modulus and of preconsolidation stress. As the factor of safety with respect to bearing capacity is usually high for foundations on granular soils, the designer is interested in the modulus, for an average applied stress limited to a value equal to about 25 to

50 % of the estimated ultimate bearing resistance. The modulus can be related to the average cone penetration resistance, q_c according to the empirical relationship given by Equation (2),

$$E_{SLS} = \alpha\, q_c \tag{2}$$

where α = an empirical coefficient. It is important to recognize that the cone penetration test (CPT) is essentially a strength test. Thus, compressibility can only be determined indirectly. However, in most granular soils, the cone penetration resistance is strongly affected by soil compressibility and stress conditions.

Test data indicate that the coefficient α varies considerably and depends on soil type and stress conditions as well as on stress level. Based on a review of results of cone penetration tests in normally consolidated sand in calibration chambers, Robertson and Campanella (1986) proposed a range for α between 1.3 and 3.0. This range agrees well with recommendation by Schmertmann (1970) for use of CPT data to analyze the settlement of isolated footings on coarse-grained soils. For overconsolidated sand, Robertson and Campanella (1986) suggest that the ratio E_{25}/q_c is approximately 3 to 6 times larger than that for normally consolidated sand (i.e. $6 < \alpha < 18$). They state that the application of higher values for overconsolidated sand should be applied with caution. Dahlberg (1975) reported tests in overconsolidated sand and found that α ranged from 2.4 through 4, increasing with increasing value of q_c. The Canadian Foundation Engineering Manual (CFEM, 1992) states that the ratio between the modulus of elasticity at 25 % of the ultimate stress, E_{25} and the cone penetration resistance, q_c is a function of soil type and compactness, as listed in Table 1.

Table 1 $\alpha = E_{25}/q_c$ from Static Cone Penetration Tests (CFEM, 1992)

Soil type	$\alpha = E_{25}/q$
Silt and sand	1.5
Compact sand	2.0
Dense sand	3.0
Sand and gravel	4.0

The α-values shown in Table 1 apply to a settlement analysis in soils that can be assumed to behave as linearly elastic media (medium stiff to stiff granular soil). The Swedish Road Authorities (Trafikverket) have issued guidelines for geotechnical design, TK Geo 11 (2011) where an empirical relationship is given for sand between elastic (Young's) modulus, E and cone penetration resistance, q_c

$$E = 4{,}3\, q_c^{0{,}93} \tag{3}$$

It is important to appreciate that values of elastic modulus determined from the above-proposed relationships are crude and should be used only for preliminary assessment of the elastic modulus.

2. SMALL-STRAIN SHEAR MODULUS

The small-strain shear modulus can be determined from seismic tests either in the field or in the laboratory. The seismic cone penetration test (SCPT) has gained increasing recognition and technical guidelines were worked out by former ISSMGE TC 10 (Geophysical Testing in Geotechnical Engineering), (Butcher et al., 2004). This guidance document describes the execution of the test and interpretation of test results. The SCPT is a relatively fast and thus cost-effective method to determine the shear wave speed of soils, c_s. The small-strain (seismic) shear modulus, G_{max} can be determined from the following relationship

$$G_{\max} = c_s^2\, \rho \tag{4}$$

where ρ is the bulk density of the soil. The small-strain shear modulus is determined at very low shear strains, typically lower than 10^{-5} (10^{-3}%). At such a low strain level, no pore pressure is generated and G_{max} reflects fundamental soil behavior independent of total or effective stress. G_{max} is an important parameter for seismic response analyses of soils. However, few geotechnical engineers appreciate that G_{max} can also be used for the solution of static geotechnical problems.

As has been pointed out by Massarsch (2004), during seismic tests at shear strain level < 10-3 %, the rate of loading (strain rate) is slow and comparable to that of static tests. This important aspect has been confirmed by a comparison of resonant column tests, performed at vibration frequencies of 30 to 35 Hz, and static torsional shear tests, Drnevich and Massarsch (1979). Shear stress was measured as a function of shear strain with high accuracy from 10^{-3}% shear strain to failure. Comparative tests by the torsional shear apparatus and the resonant column device yield almost identical values of Gmax. As resonant column tests are typically performed at 30 Hz and torsional shear tests are performed at 0.1 Hz, this indicates that at small strains (< 0.001 %) the stress-strain behavior (G_{max}) is independent of the rate of loading. For practical purposes, the effect of strain rate on medium dense and dense granular soils can be neglected up to a strain level of approximately 0.1%.

The results of a resonant column on an undisturbed, reconsolidated sample of clayey sand is shown in Fig. 1, Drnevich & Massarsch (1979). Below 10^{-3}% shear strain the shear modulus appears to be unaffected by shear strain (and thus strain rate). However, when shear strains exceed 10^{-3}%, the shear modulus decreases. At 10^{-1}% shear strain, the shear modulus of the clayey sand is only about 30% of the maximum value.

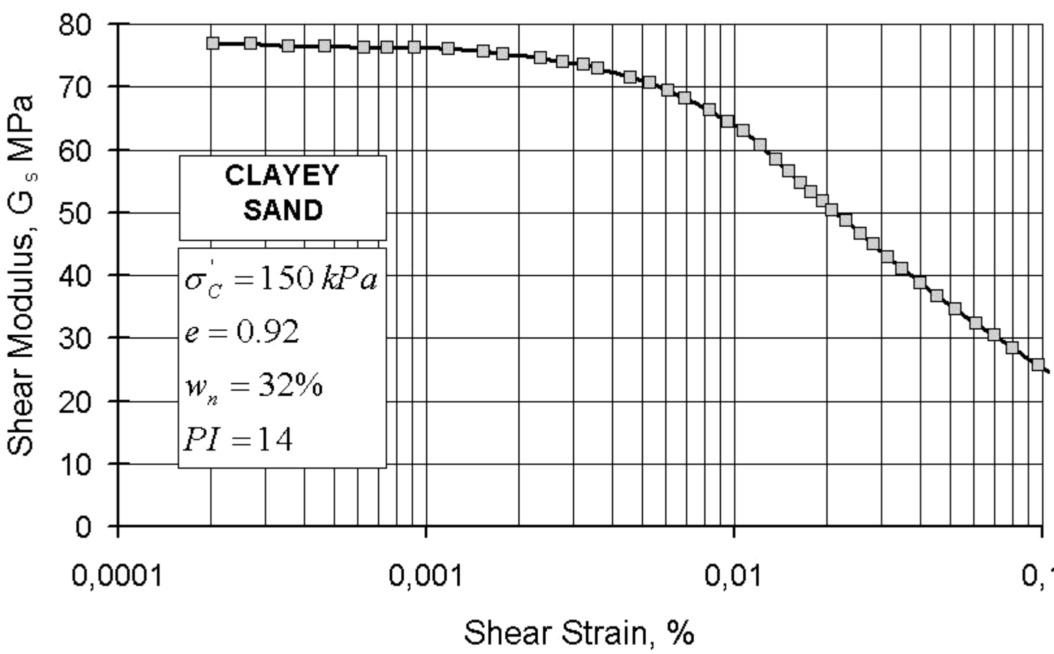

Figure 1. Variation of shear modulus with shear strain determined from torsional resonant column test, after Drnevich & Massarsch (1979).

Based on extensive resonant column tests, Hardin (1978) suggested that the small-strain shear modulus, G_{max} of sand can be estimated from the following relationship

$$G_{max} = \frac{625}{0.3 + 0.7e^2}\left(\sigma'_m\,\sigma_r\right)^{0.5} \tag{5}$$

where: e = void ratio, σ'_m = mean effective stress and σ_r = reference stress (100 kPa). The mean effective stress σ'_m is defined as

$$\sigma'_m = \sigma'_v\left(\frac{1 + 2K_0}{3}\right) \tag{6}$$

where: σ'_v = vertical effective stress, K_0 = coefficient of lateral earth pressure at rest. Even if the horizontal stress (and thus K_0) are not known, it is preferable to estimate the coefficient of horizontal earth pressure at rest, K_0 based on engineering judgment than to neglect the significance of horizontal effective stress. Hardin (1978) found that for granular soils the overconsolidation ratio, *OCR* has little or no influence on G_{max}. In Figure 2, the variation of the small-strain shear modulus, G_{max} is shown for different values of the void ratio, *e* as a function of the mean effect stress, σ'_m. It has been assumed that the ground water is at the ground surface, the coefficient of lateral earth pressure at rest, K_0 = 0.5 and the bulk density, ρ = 2000 kg/m^3.

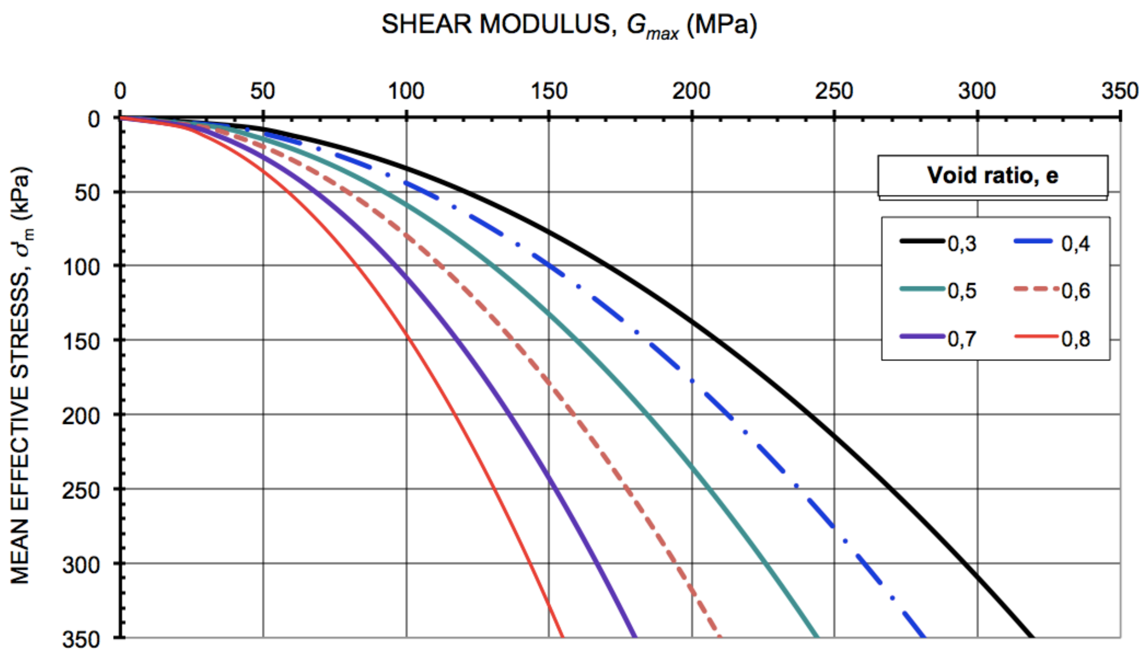

Figure 2. Variation of the small-strain shear modulus with mean effective stress for different values of void ratio, cf. Equation (5). The ground water level is assumed at the ground surface.

3. STATIC SHEAR MODULUS

3.1 Modulus Degradation of Fine-grained Soils

Figure 1 shows that the shear modulus decreases with increasing shear strain level. The static shear modulus, *G* of sand can be estimated from the following relationship

$$G = R_M \, G_{\max} \tag{7}$$

where: R_M = modulus reduction factor, G_{max} = shear modulus at small strain (<10^{-3}%). Most earth structures or foundations have at working load a factor of safety, *FS* > 1,5.

The modulus reduction factor, R_M of fine-grained soils has been investigated by Massarsch (2004). Based on the evaluation of extensive resonant column test data, a relationship was found which describes the variation of the normalized shear modulus is shown as a function of shear strain, for different values of *PI*, Figure 3. It is apparent that shear modulus degradation increases with decreasing plasticity index, *PI*.

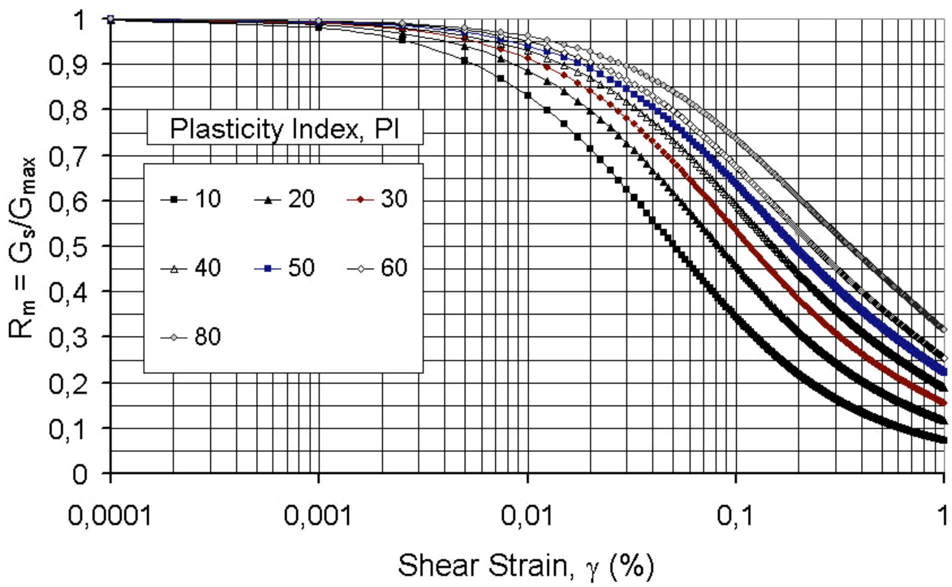

Figure 3. Variation of the normalized shear modulus as a function of shear strain for different values of *PI*, Massarsch (2004).

3.2 Modulus Degradation of Granular Soils

Little information has been published on how plasticity index, *PI*, void ratio, *e* and degree of saturation, S_r affect the modulus reduction factor, R_M of silts and sands. Hardin (1972) published results of resonant column test, showing the degradation with shear strain of the shear modulus of sandy and silty soils. The original test data reported by Hardin were re-interpreted to obtain values of the modulus reduction factor, R_M at 0.5 % shear strain. The *PI* of the investigated soil samples varied between 0 and 22 %, *e* between 0.33 and 0.77 and S_r between 0 and 100 %, cf. Table 2.

Table 2. Modulus reduction factor R_M at 0.5 % shear strain for granular soils, based on data provided by Hardin (1972).

$R_M = G/G_{max}$	Plasticity Index, *PI*	Void Ratio, e	Degree of Saturation S_r	Sample Description
-	%	-	%	-
0.1	0,01	0.62	0,01	WES Sand
0.09	0,01	0.62	100	St. Johns Sand
0,11	6	0.57	33	Airforce Silty Sand 4
0.2	13	0.63	91	Airforce Silty Clay
0.1	6	0.67	73	Vicksburg Loess
0.14	11	0.59	91	Vanceburg
0.11	9	0.69	44	Allen
0.12	9	0.60	100	Kentucky 55
0.15	4	0.42	69	West Virginia Shale
0.2	22	0.77	94	Virginia Clay
0.09	0,01	0.33	86	Dover
0.13	0,01	0.63	47	Prestonsburg
0.145	0,01	0.71	40	Kirtland

The dependence of the modulus reduction factor, R_M on void ratio, e is shown in Figure 4. In spite of some scatter in the data, it is apparent that modulus reduction is more pronounced in dense (low void ratio) than in loose (high void ratio) granular soils.

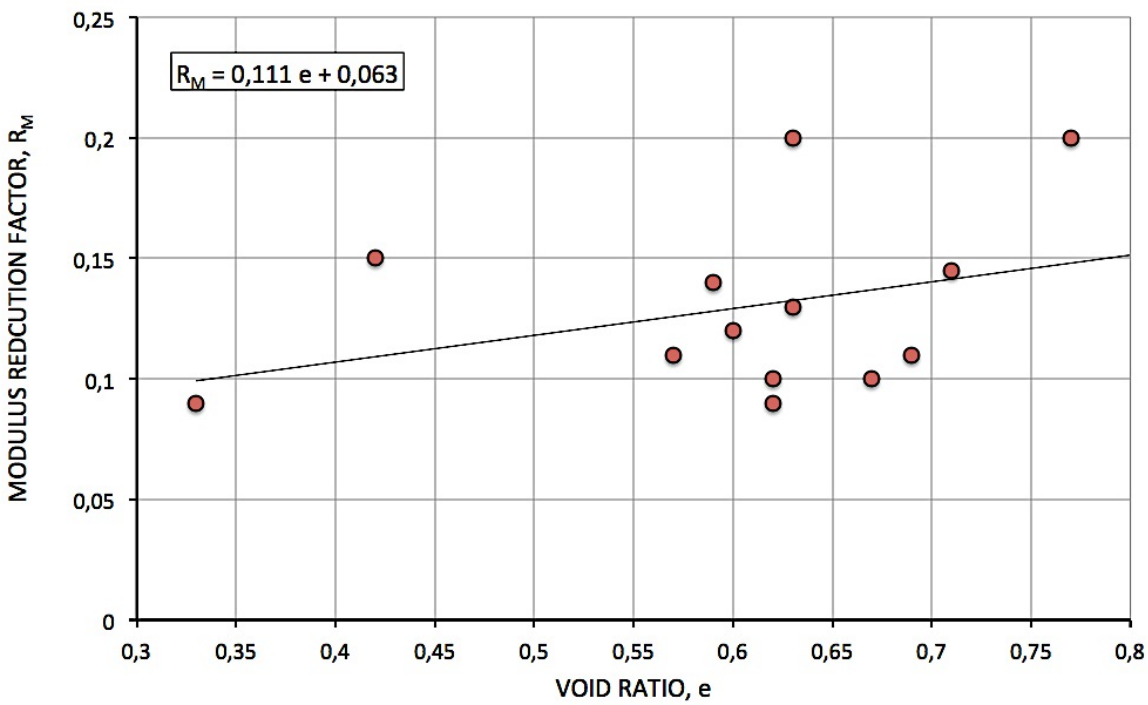

Figure 4. Variation of modulus reduction factor R_M at 0.5 % shear strain with void ratio, cf. Table 2.

From Figures 4, a relationship between the void ratio, e and the modulus reduction factor, R_M is obtained

$$R_M = 0.111\,e + 0.063 \tag{8}$$

The modulus reduction factor for a void ratio between e = 0.3 and 0.8 varies between R_M = 0.096 and 0.152. An average R_M -value for medium dense (compact) sand with a void ratio e = 0.60 would be R_M = 0.13. The relative density of sands can be approximately characterized by the following ranges of void ratio; cf. Table 3.

Table 3. Approximate range of values for void ratio in sand with different densities.

Density	Void Ratio, e
Very dense	0,35 – 0,45
Dense	0,45 - 0,55
Compact	0,55 – 0,65
Loose	0,65 – 0,75
Very loose	0,75 – 0,85

The dependence of the modulus reduction factor on the degree of saturation is shown in Figure 5. There is scatter in the data at high degree of saturation but the trend shows that S_r has only a slight effect on R_M. The lower bound of R_M is 0.1.

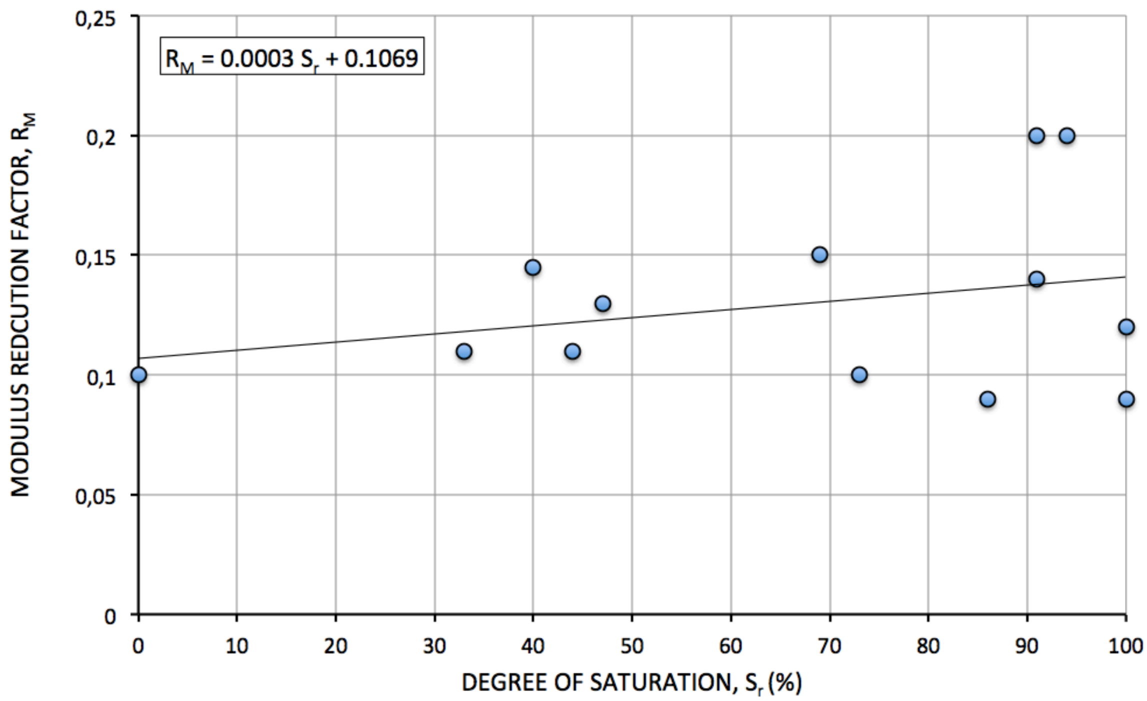

Figure 5. Variation of modulus reduction factor R_M at 0.5 % shear strain with degree of saturation, cf. Table 2.

From Figures 5, the effect of the degree of saturation on the modulus reduction factor can be determined from the following equations

$$R_M = 0.0003\, S_r + 0.1069 \tag{9}$$

There appears to be a slight increase of the modulus reduction factor with increasing degree of saturation. For most soils the average value of R_M can be assumed to be 0.13, a value similar to that of the void ratio. However, for most practical purposes, the influence of S_r on R_M can be neglected.

A relationship between the modulus reduction factor at 0.5 % shear strain for fine-grained soils has been proposed by Massarsch (2004). These data have been combined with the data presented in Table 2. The relationship between modulus reduction factor, R_M at 0.5 % shear strain and plasticity index, *PI* of all tested soils is shown in Figure 6 for *PI* values ranging from 0 to 100 %.

A relatively simple relationship can be found, defining the relationship between the modulus reduction factor, R_M at 0.5 % shear strain and plasticity index, *PI* is obtained

$$R_M = 0.0043\, PI + 0.103 \tag{10}$$

In spite of the wide range of soils, a surprisingly good correlation between the modulus reduction factor, R_M and plasticity index, *PI* exists at a shear strain level of 0.5 %. For most granular soils with a plasticity index of 5 %, the modulus reduction factor is approximately 0.13.

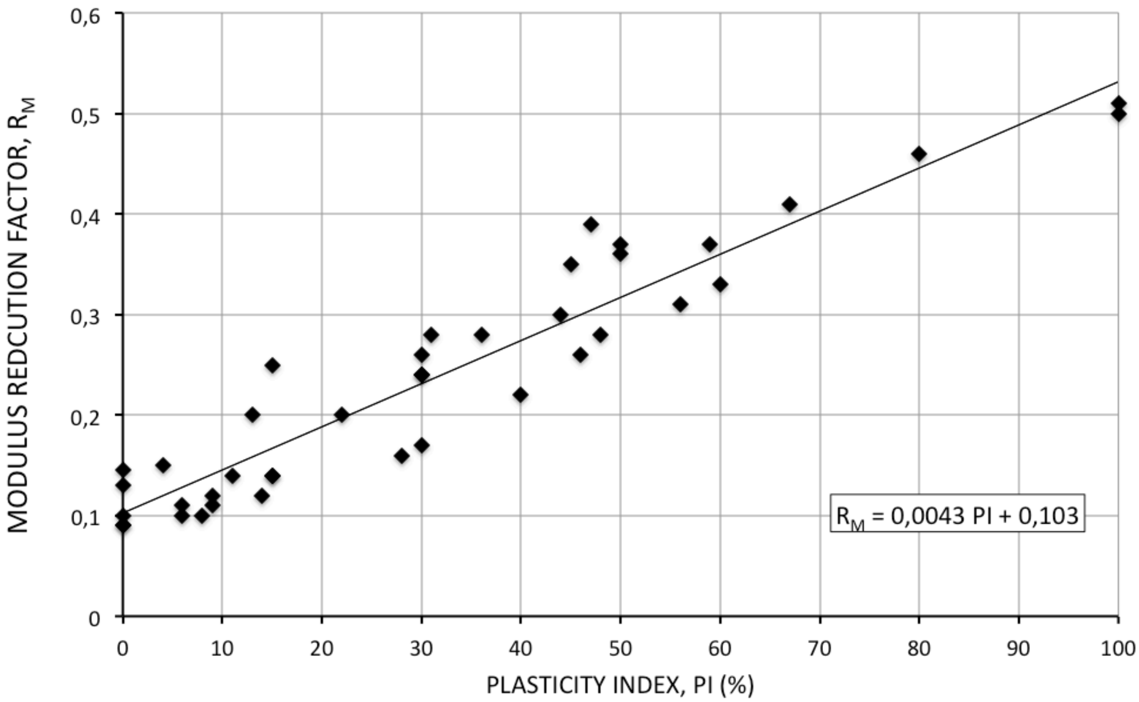

Figure 6. Relationship between modulus reduction factor, R_M and plasticity index, *PI* including data from Table 2 and results by Massarsch (2004).

3.3 Relationship between Moduli

From the shear modulus, *G* it is possible to determine the elastic (Young's) modulus, *E*, and the constrained modulus, *M*, respectively if Poisson's ratio, ν is known

$$E = 2(1+\nu)\,G = 2(1+\nu)\,R_M G_{\max} \tag{11}$$

$$M = \frac{(1-\nu)}{(1-2\nu)(1+\nu)} E = \frac{2(1-\nu)}{(1-2\nu)} G \tag{12}$$

The ratio between different moduli for a range of values of Poisson's ratio, ν is shown in Table 4.

Table 4. Modulus ratio for different values of Poisson's ratio.

Poisson's ratio	E/G	M/G	M/E
0,25	2,50	3,00	1,20
0,30	2,60	3,50	1,35
0,33	2,66	3,94	1,48
0,40	2,80	6,00	2,14
0,49	2,98	51,00	17,11

For granular soils, the elastic modulus is about 2.5 to 2.7 times the shear modulus. If for granular soils ν = 0,30 is assumed, the elastic modulus *E* = 2.6 *G* and the confined modulus *M* = 3.5 *G*. The variation of the elastic modulus, *E* as a function of mean effective stress can be determined by substituting Equation (5) into Equation (11). Based on the results presented in the preceding chapter, for sand an average value R_M = 0.13 was chosen. Assuming the ground water table at the ground surface, Poisson's ration, ν = 0.30, unit weight ρ = 2000 kg/m^3 and coefficient of lateral earth pressure at rest, K_0 = 0.5, the elastic modulus, E at 0.5% shear strain (i.e. "static modulus") can be determined. The variation of elastic modulus as a function of mean confining stress and for different values of void ratio is shown in Figure 7.

Applying a similar approach, the constrained modulus, M can be determined from Equation (12). The variation of the constrained modulus as a function of mean confining stress for different values of void ratio has been determined and is shown in Figure 8. The same assumptions were made regarding soil properties as in Figure 7.

4. STRESS ADJUSTMENT OF CONE RESISTANCE

The mean effective confining stress, σ'_m influences the cone penetration resistance and relative density. After adjusting the cone resistance and relative density with respect to the mean effective confining stress it is possible to obtain relationships, which are stress-independent. This concept will be illustrated in the following paragraphs.

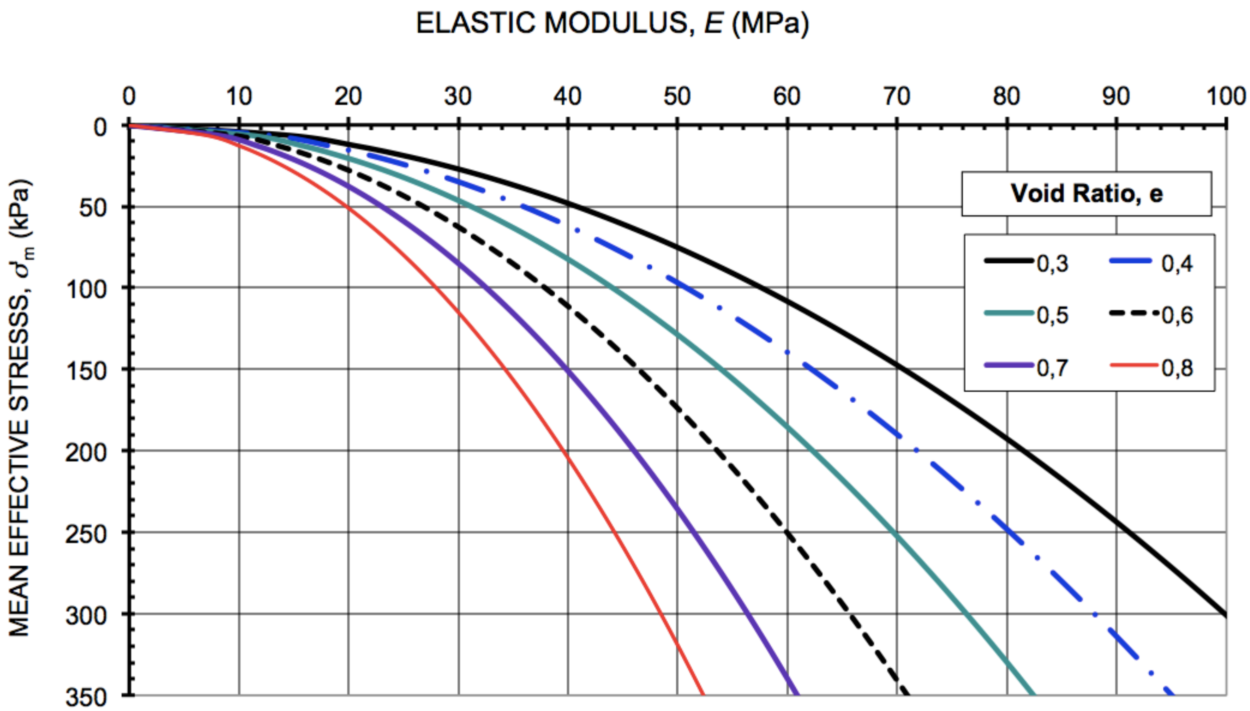

Figure 7. Dependence of elastic modulus, E at 0.5% shear strain on mean confining stress as a function of void ratio, e.

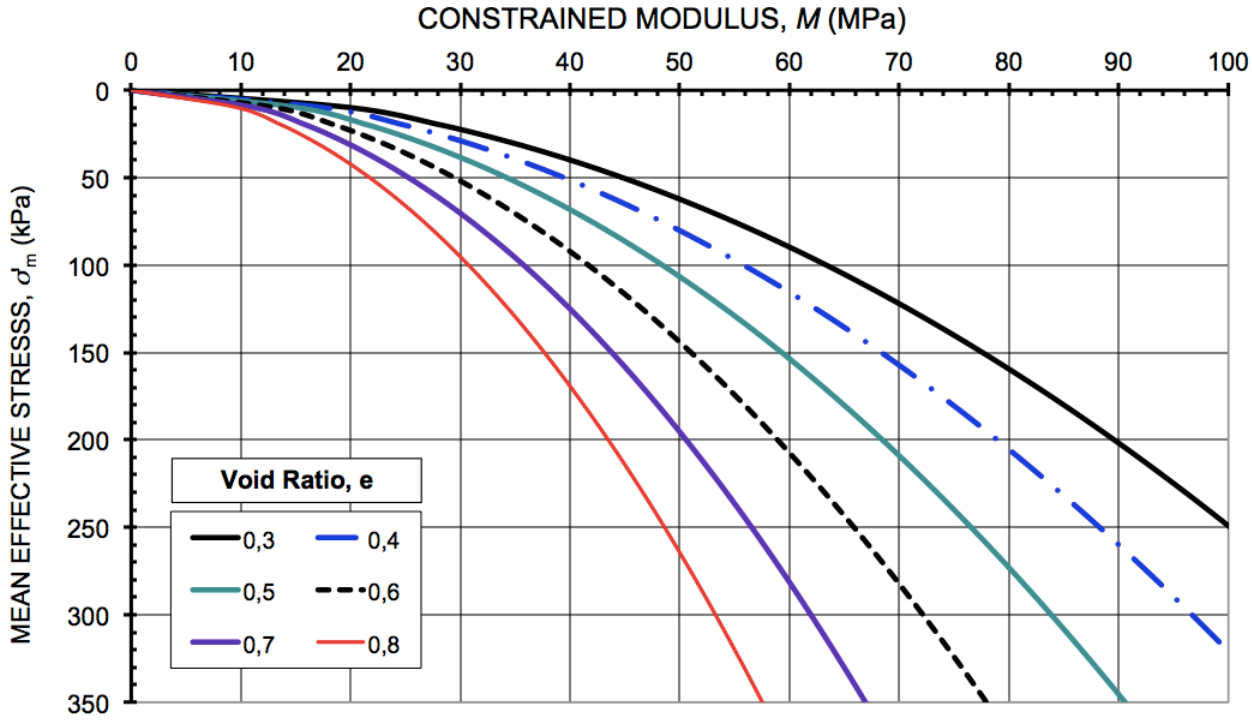

Figure 8. Dependence of constrained modulus M at 0.5 % shear strain on mean confining stress as a function of void ratio.

4.1 Cone Penetration Test

Based on a review of extensive field data, Massarsch (1994) proposed a stress adjustment factor, C_M to take into account the effect of mean effective stress σ'_m on the cone penetration resistance

$$C_M = \left(\frac{\sigma_r}{\sigma'_m}\right)^{0,5} \tag{13}$$

where C_M = stress adjustment factor ≤ 2.5; σ_r = reference stress = 100 kPa and σ'_m = mean effective stress. Note that at a mean confining stress of 100 kPa, C_M = 1. It is now possible to determine the stress-adjusted cone penetration resistance q_{cM}

$$q_{cM} = q_c\, C_M = q_c \left(\frac{\sigma_r}{\sigma'_m}\right)^{0,5} \tag{14}$$

The application of the stress-adjusted cone penetration resistance for the determination of the compression modulus of granular soils will be discussed below.

4.2 Relative Density

In geotechnical design, the relative density, D_r (or Density Index, *ID*) is frequently used to express the state of density of granular soils. The definition of the density index, $ID = (e_{max} - e)/(e_{max} - e_{min})$ is based on the assumption that the void ratio of the soil can be reliably determined for the "maximum" and "minimum" density of a natural soil. However, relative density is an ambiguous and qualitative expression, which, whenever possible, should be avoided.

Over the years, "*relative density*" has been correlated to other geotechnical parameters of granular soils. Baldi et al. (1986) have proposed, based on extensive pressure chamber tests, the following relationship between cone penetration resistance, q_c, the mean effective stress, σ'_m and the relative density, D_r, cf. Figure 9.

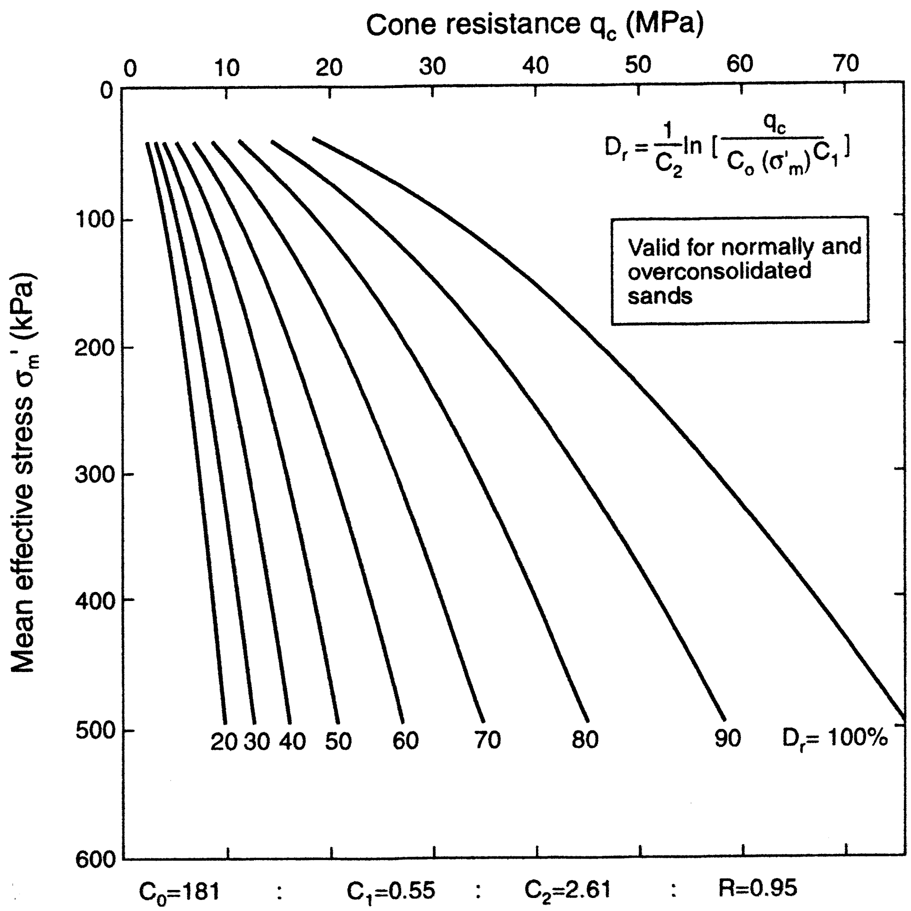

Figure 9. Relationship between cone resistance, mean effective stress and relative density according to Baldi et al. (1986).

Equation (15) gives the equation for the best fit of cone penetration resistance q_c, relative density, D_r and mean effective stress σ'_m

$$D_r = \frac{1}{C_2} \ln\left[\frac{q_c}{C_0(\sigma'_m)^{C_1}}\right] \tag{15}$$

with the following coefficients: C_0 = 181; C_1 = 0.55 and C_2 = 2.61. By rearranging terms, Equation (15) can be expressed in exponential form

$$q_c = C_0\left(\sigma'_m\right)^{C_1} e^{C_2 D_r} \tag{16}$$

Substituting the coefficients yields the following relationship

$$q_c = 181\,(\sigma'_m)^{0.55}\, e^{2.61\,D_r} \tag{17}$$

Note that in (16) and (17), e is not the void ratio but the numerical constant for the natural logarithm. Also, the coefficients were determined experimentally and are not dimensionless. This aspect is not always appreciated. Substituting q_{cM} from Equation (14) into Equation (17) and rearranging terms yields

$$q_{CM} = 181\,(\sigma_m^{'})^{0.55}\, e^{2.61 D_r} (\frac{\sigma_r}{\sigma_m^{'}})^{0.5} \tag{18}$$

Introducing a correction factor for the exponent, 0.55 of mean effective stress in Equation (18), a simple relationship between the stress-adjusted cone penetration resistance, q_{cM} and relative density, D_r is obtained

$$q_{cM} = 2350\, e^{2.61 D_r} \tag{19}$$

Substituting Equation (14) into Equation (19) yields

$$q_c = 2350\, e^{2.61 D_r} \left(\frac{\sigma_m^{'}}{\sigma_r}\right)^{0.5} \tag{20}$$

Figure 10 compares the relationship suggested by Baldi (1986) with the relationship given by ((20).

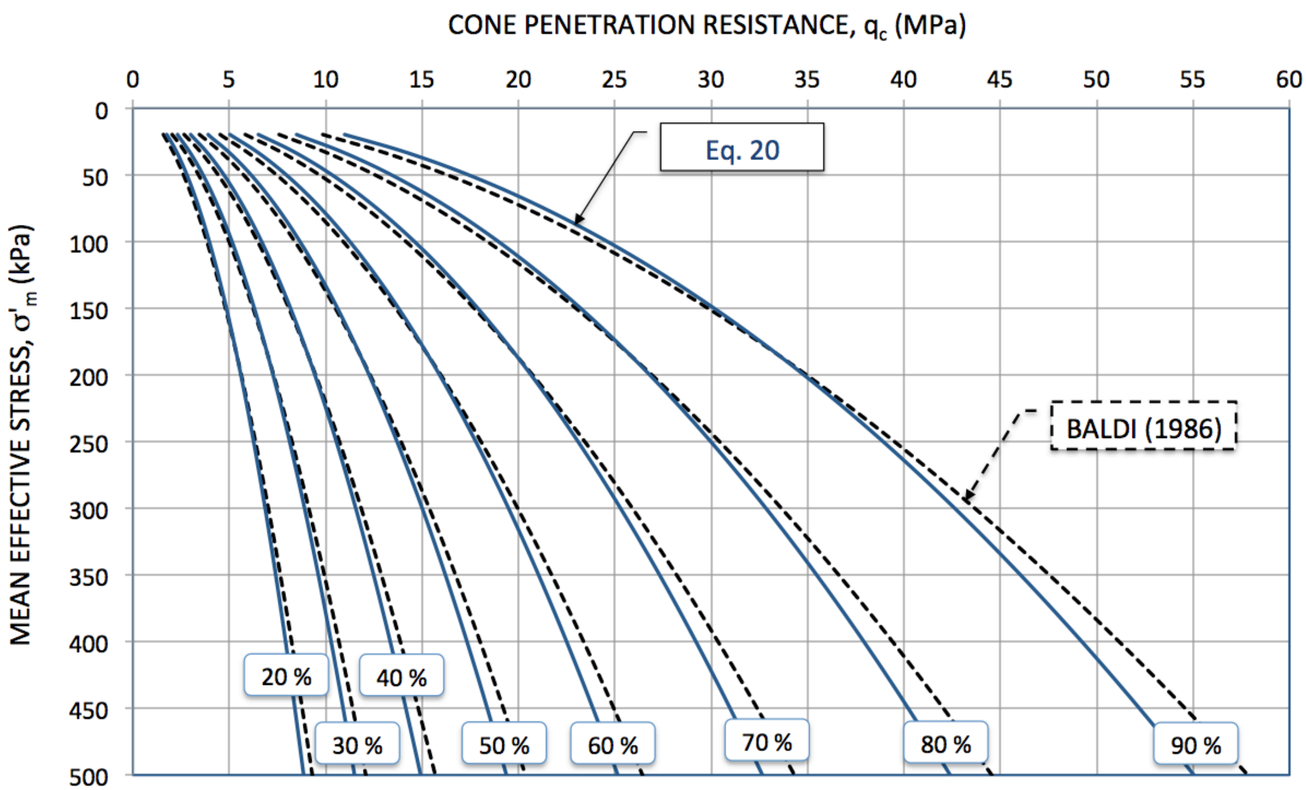

Figure 10. Comparison between relationships proposec by Baldi (1986), Fig. 9 and Equation (20).

It is worthwhile noting that by adjusting the cone penetration resistance with respect to the mean effective stress, the curves shown in Figure 9 collapse into a simple relationship given by Equation (19).

5. RELATIONSHIP BETWEEN SOIL MODULUS AND CONE PENETRATION RESISTANCE

5.1 Tangent Modulus

The tangent modulus concept is widely used for determining settlements. For details about the application of the tangent modulus method, reference is made to the Canadian Foundation Engineering Manual, CFEM (1992). The tangent modulus, M_t can be derived from the following expression

$$M_t = m\,\sigma_r \left(\frac{\sigma'}{\sigma_r}\right)^{1-j} \tag{20}$$

where: m = modulus number, σ_r = reference stress (100 kPa), σ' = applied effective stress, and j = stress exponent. For cohesionless soil (silt, sand and gravel), the stress exponent is larger than zero, $j > 0$. Integrating Equation (20) yields

$$\varepsilon = \frac{1}{mj}\left[\left(\frac{\sigma_1'}{\sigma_r}\right)^{j} - \left(\frac{\sigma_0'}{\sigma_r}\right)^{j}\right] \tag{21}$$

Fellenius (2011) has discussed the practical application of the tangent modulus concept in detail and reference is made for detailed information. Typical values of the stress exponent j and modulus number m for different materials are given in the CFEM, cf. Table 5.

Table 5. Typical values of stress exponent j and modulus number *m*, CFEM (1992).

SOIL OR ROCK TYPE			STRESS EXPONENT (j)	MODULUS NUMBER (m)
Rock:	high strength		1	1 000 000 - 1000
	low strength		1	1 000 - 300
Tills:	very dense to dense		1	1 000 - 300
Gravel			0.5	400 - 40
Sand:	dense		0.5	400 - 250
	compact		0.5	250 - 150
	loose		0.5	150 - 100
Silt:	dense		0.5	200 - 80
	compact		0.5	80 - 60
	loose		0.5	60 - 40
Clays:	silty clay and	hard	0	60 - 20
	clayey silt	stiff, firm	0	20 - 10
		firm, soft	0	10 - 5
	soft marine clays and organic clays		0	20 - 5
Peat			0	5 - 1

5.2 Dense sand: $j = 1$

In very dense sand, the stress exponent $j = 1$. Vertical effective stress prior to, and after imposing an axial load are defined by σ'_0 and σ'_1, respectively. It is assumed that compression occurs without lateral displacement. By inserting these values and the reference stress, σ_r, which is equal to 100 kPa, Equation (21) becomes:

$$\varepsilon = \frac{1}{100m}\left(\sigma_1^{'} - \sigma_0^{'}\right) = \frac{1}{100m}\Delta\sigma^{'} \tag{22}$$

where: $\Delta\sigma' = \sigma'_1 - \sigma'_0$. Considering the relationship between strain, ε, change of effective stress, $\Delta\sigma'$ and soil modulus, M

$$\varepsilon = \frac{\Delta\sigma^{'}}{M} \tag{23}$$

the following simple relationship is obtained between modulus number, m and confined modulus, M

$$m = \frac{M}{100} \tag{24}$$

or

$$M = 100\ m \tag{25}$$

The elastic modulus can now be estimated using Equation (12) or Table 5. For granular soils with Poisson's ratio $\nu \approx 0.3$, the modulus ratio $M/E = 1.35$, from which the relationship between the elastic modulus, E and the modulus number, m for dense sand is obtained

$$E = 74\ m \tag{26}$$

Thus, for dense (heavily overconsolidated) sand, the modulus is independent of effective stress and can be readily estimated if the modulus number is known. For very dense sand, gravel and till, the modulus number varies typically between $300 < m < 1000$, cf. Table 5. The elastic modulus according to Equation (26), is in the range of 22 MPa < E < 74 MPa.

5.3 Sandy Soils: j = 0,5

Medium dense and loose sand can be assumed to be normally consolidated. Substituting j = 0,5 into ((21) yields the following relationship, which depends on the stress interval between vertical effective stress prior to loading, σ'_0 and after loading, σ'_1.

$$\varepsilon = \frac{1}{5m}\left[\left(\sigma_1^{'}\right)^{0,5} - \left(\sigma_0^{'}\right)^{0,5}\right] \tag{27}$$

Note that in Equation (27), σ'_0 corresponds to the vertical effective stress prior to loading and σ'_1 to the vertical effective stress after load has been imposed. Equation (27) is affected by the choice of units, as reference stress σ_r = 100 kPa was chosen. Substituting Equation (23) into Equation (27) yields

$$M = 5m\,\frac{\left(\sigma_1^{'}\right) - \left(\sigma_0^{'}\right)}{\left(\sigma_1^{'}\right)^{0,5} - \left(\sigma_0^{'}\right)^{0,5}} \tag{28}$$

The deformation modulus of sandy soils is thus non-linear and depends on the applied stress level. In order to determine a modulus value for a stress interval, it is assumed that the stress level increases by a factor of 2

$$\sigma_1^{'} = 2\sigma_0^{'} \tag{29}$$

Now the confined modulus, *M* can be determined as function of the modulus number, *m* and vertical effective stress, σ'_v

$$M = 12\, m\left(\sigma_v^{'}\right)^{0.5} \tag{30}$$

Substituting *M/E* = 1.35 for sandy soil (assuming ν = 0,3) the following relationship between elastic modulus and modulus number is obtained

$$E = 8.89\, m\left(\sigma_v^{'}\right)^{0.5} \tag{31}$$

Typical values of the modulus number, *m* for sandy soils are given in Table 6.

Table 6. Typical values of modulus number for sand.

Sand Type	Modulus Number, *m*
Dense	250 – 400
Compact	150 – 250
Loose	100 – 150

Substituting these values in Equation (31) gives a range of values for the elastic modulus for different densities of sand, cf. Table 7. The elastic modulus is thus dependent on the vertical effective stress, an aspect, which is not taken into account in empirical correlations, cf. Equation (2).

6. MODULUS DETERMINED FROM CPT

Based on the concept of stress-adjusted cone penetration resistance, q_{CM} which has been described above, Massarsch (1994) proposed a semi-empirical method to estimate the modulus number *m* for sands based on the cone penetration test

$$m = a\left(\frac{q_{cM}}{\sigma_r}\right)^{0.5} \tag{32}$$

where *a* is a modulus modifier, which has been determined from the evaluation of extensive field and laboratory tests. The parameter *a* varies within a relatively narrow range for each soil type. Massarsch and Fellenius (2002) proposed the following values of *a* for sand as listed in Table 7.

Table 7. Typical values of elastic modulus number for sand.

Sand Type	Modulus Modifier, *a*
Silty loose	20
Loose	22
Compact	28
Dense	35

The elastic modulus for dense (overconsolidated) granular soils can be determined if Equation (32) is substituted into Equation (26)

$$E = 74\, a\left(\frac{q_{cM}}{\sigma_r}\right)^{0.5} \tag{33}$$

If according to Table 7, a = 35, and σ_r = 100 kPa, the following relationship between the E-modulus and the stress-adjusted cone penetration resistance is obtained

$$E = 259\left(q_{cM}\right)^{0.5} \tag{34}$$

The elastic modulus for loose to compact granular soils can be determined by substituting Equation (32) into Equation (31)

$$E = 8.89\, a\left(\frac{q_{cM}}{\sigma_r}\right)^{0.5} \tag{35}$$

Equation (35) is affected by the choice of units, as reference stress σ_r = 100 kPa. If for medium dense sand according to Table 7, an average value of the modulus modifier a = 25 is chosen, and substituting σ_r = 100, the following relationship between the E-modulus and the stress-adjusted cone penetration resistance is obtained

$$E = 22\left(q_{cM}\,\sigma_v^{'}\right)^{0.5} \tag{36}$$

Note that all values must be inserted using the same units (kPa). The relationship given by Equation (36) is shown in Fig. 10. The vertical effective stress is an important factor which influences the elastic modulus, when determined based on cone penetration tests. In Figure 10, also the relationship given by Equation (3) is shown. For medium dense to loose soils with a cone penetration resistance 5 < q_c < 10, Equation (3) appears to agree reasonably well with average values of vertical effective stress.

Figure 10 demonstrates that simple, empirical relationships between cone penetration resistance and elastic modulus can significantly underpredict the elastic modulus, cf. Table 1.

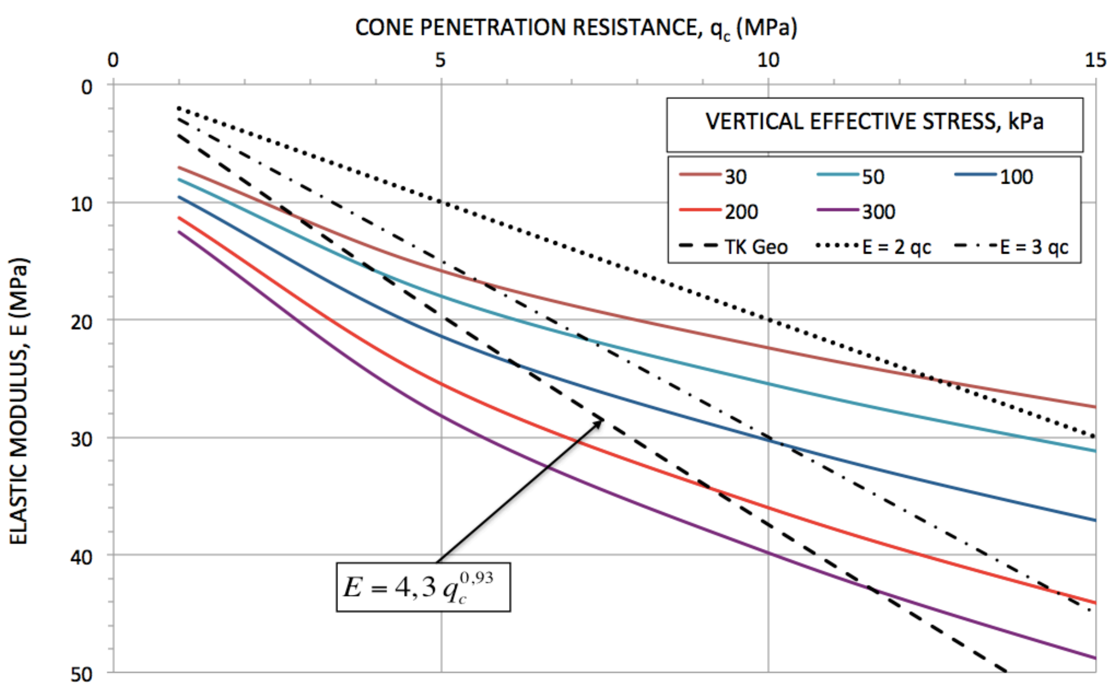

Figure 11. Relationship in loose to medium dense sand between cone penetration resistance and elastic modulus for different values of vertical effective stress. Also indicated are empirical relationships from Equation (32) and Equation (2) and Table 1, respectively.

6. CONCLUSIONS

Although geotechnical design requires the determination of settlement at working load (SLS), little information regarding the deformation properties of granular soils can be found in the geotechnical literature.

Two methods are presented which can be used to estimate the soil modulus: a) based on small-strain shear modulus and b) based on cone penetration resistance.

The shear modulus of granular soils is strongly affected by shear strain. Results of extensive resonant column tests have been reviewed. The modulus reduction factor, R_M at 0.5 % shear strain is used to describe deformation at working load (SLS). The modulus reduction factor is strongly influenced by the plasticity index. In sandy soils, R_M ranges typically between 0.10 and 0.15 and is lower in a dense state (low void ratio) than in a loose state (high void ratio). The elastic modulus, E and the confined modulus, M can be estimated based on the shear modulus, G.

Stress adjustment of the cone penetration resistance to a reference stress (100 kPa) facilitates data interpretation. The relatively complex relationship between cone penetration resistance and relative density can be simplified by applying the stress adjustment concept. It results in a simple relationship between relative density, D_r and stress-adjusted cone penetration resistance, q_{cM}.

The tangent modulus method concept can be used to assess the deformation modulus of granular soils. A method is proposed which permits the estimate of the modulus number for different types of sandy soils, based on the stress-adjusted cone penetration test, q_{cM}. The modulus of dense sand can be estimated based on a modulus number, m which either can be determined from empirical relationships or from stress-adjusted cone penetration tests. A similar concept is proposed for loose and medium dense sand.

Simple relationships can be used to estimate the elastic modulus of loose, medium dense and dense sand based on stress-adjusted cone penetration resistance. The elastic modulus is affected by the cone penetration resistance and the vertical effective stress.

A comparison with empirical relationships in the geotechnical literature suggest that these relationships underpredict the elastic modulus.

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