Recent developments in vibratory driving and soil compaction

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ABSTRACT. During the past two decades, the capacity and performance of hydraulic vibrators has undergone a rapid development. By the use of vibrators with variable frequency and eccentric moment, the transmission of the vibration energy to the surroundings can be controlled. Methods and empirical rules for estimating the drivability of sheet piles are presented. The application of modern vibrators is not limited to the driving of piles and sheet piles. The efficiency of vibratory compaction can be enhanced by adapting the vibrator to the resonance frequency of the vibrator-pile-soil system. The optimization of the compaction process by electronic process control is highlighted and illustrated. A new concept is proposed where the dynamic penetration resistance is correlated to the pile penetration speed, measured in terms of the number of vibration cycles during the driving. The validity of the concept is demonstrated by a case history. Finally, examples of pile and ground improvement solutions are presented which take advantage of the fundamental aspects of vibratory driving.

1. INTRODUCTION

Modern, hydraulic vibrators can be used for different construction processes, such as the installation and/or extraction of piles and sheet piles and compaction . However, in spite of their frequent use in the foundation industry, many aspects of vibratory driving are still based on overly simplistic empirical rules. There is a need for a better understanding of the parameters of importance for vibratory driving of piles and sheet piles. When using vibrators for pile and sheet pile driving or vibratory compaction of granular soils, the following issues need to be considered:

- drivability: select appropriate vibrator type and capacity to assure efficient installation
- bearing capacity: verify that the required bearing capacity can be achieved
- vibratory compaction: assure that the compaction requirements can be achieved
- environmental impact: minimize emission of ground vibrations and/or noise.

The four issues are closely interrelated and one cannot be addressed without considering the other three.

2. VIBRATOR PERFORMANCE

2.1 Fundamentals

The performance of modern vibrators can be controlled and adapted for optimal performance. Several parameters govern the vibratory driving process and it is important to realize how they can be varied to achieve efficient driving or compaction. Vibratory excitation affects a pile in a different way than does impact driving. An impact hammer imparts a short duration blow to the pile head, sending a stress wave down the pile. The initial hammer energy is reduced in passing through the pile helmet and pile cushion. Each blow must be able to activate the inertia of the pile and overcome the static soil resistance along the pile shaft and at the pile toe. In contrast, in the case of vibratory driving, the pile is rigidly connected to the vibrator, resulting in minimal energy loss. The vibration frequency is relatively slow, typically below 40 Hz (2,400 rpm). Thus, the wave length is much longer than in the case of impact driving. The pile is kept in an oscillating motion during the entire vibratory driving process. In the case of vibratory driving in coarse-grained soil, the toe resistance is approximately similar to that during impact driving, while the shaft resistance is usually lower (Westerberg et al. 1995). It is generally recognized that vibratory driving is most effective in coarse-grained soil and less efficient in fine-grained (cohesive) soils.

The most important parameters of the vibrator that govern the drivability of piles and sheet piles, are the eccentric (sometimes called "*static*") moment, the vibration frequency, and the mass ratio between the vibrator and pile. These parameters will be discussed briefly in this section. Modern vibrators are hydraulically driven, which allows continuous variation of the vibrator frequency during operation. The vertical oscillation of the vibrator is generated by counter-rotating eccentric masses. The peak value of the centrifugal force, F_v, acting in the vertical direction, depends on the eccentric (static) moment, M_e, and on the circular frequency ϖ ($2\pi f$), of the rotating eccentric masses.

$$F_V = M_e\,\omega^2 \tag{1}$$

Many modern vibrators also enable the centrifugal force to be adjusted continuously during operation, resulting in a variable displacement amplitude, s. This feature allows the machine operator to start up and shut down the vibrator at zero vibration amplitude, thereby reducing the risk of vibration amplification due to resonance effects. An additional important factor for the vibrator performance is the displacement amplitude, which together with the centrifugal force, F_v, determines the driving ability of the vibrator. For a vibrator suspended above the ground surface, the axial displacement amplitude (single amplitude), s can be determined from the following relationship.

$$s = \frac{M_e}{M_d} \tag{2}$$

The "*total dynamic mass*", M_d, is the sum of all masses, which need to be accelerated by the vibrator. This includes, e.g., the mass of vibrator (but not the static mass separated by soft springs from the vibrator), the pile, and the vibrator clamp. Note that most equipment manufacturers express the displacement amplitude as peak-to-peak ("*double*") amplitude. From Eq. (2), it can be appreciated that the free pile displacement amplitude, s, is independent of the vibration frequency, f. In order to maximize the displacement amplitude, the total dynamic mass, M_d, should be kept as small as possible.

2.2 Vibrator Performance Characteristics

Before 1980, most hydraulic vibrators had fixed eccentric moment, with a typical operating frequency between 20 and 30 Hz. These vibrators were used for basic construction works, such as driving and extracting of sheet piles. Table 1 shows a typical range of performance parameters for vibrators with fixed eccentric moment.

TABLE 1. Müller Vibrators with fixed eccentric moment (Müller-ThyssenKrupp 2004).

Vibrator	Units	MS-25 H2	MS-25 H3	MS-50 H2	MS-50 H3
Centrifugal force	kN	774	774	1430	1430
Eccentric moment	kgm	25	25	50	50
Speed	rpm	1680	1680	1615	1615
Frequency	Hz	28	28	26.9	26.9
Static pulling force	kN	400	400	500	500
Dynamic mass[1)]	kg	1930	2550	3340	3820
Total mass[1)]	kg	3200	3600	6300	6790
Vibration (double) amplitude[1)]	mm	25.9	19.6	29.9	26.2

[1)] without clamping device/pile

Vibrators have experienced a rapid development in terms of power, range of operating parameters (eccentric moment and frequency), and monitoring of the driving and extraction process. An important advantage of hydraulic vibrators over traditional impact hammers (i.e., not underwater hydraulic impact hammers) is that they can be operated under water. A major development in vibrator design came with the introduction of a stepwise adaptation of the eccentric moment according to specific driving requirements. Such change of the eccentric moment is made manually by adding or removing weights, Figure 1. For example, if high frequency operation is required, weights are removed on-site to increase to the desired frequencies with the same centrifugal force.

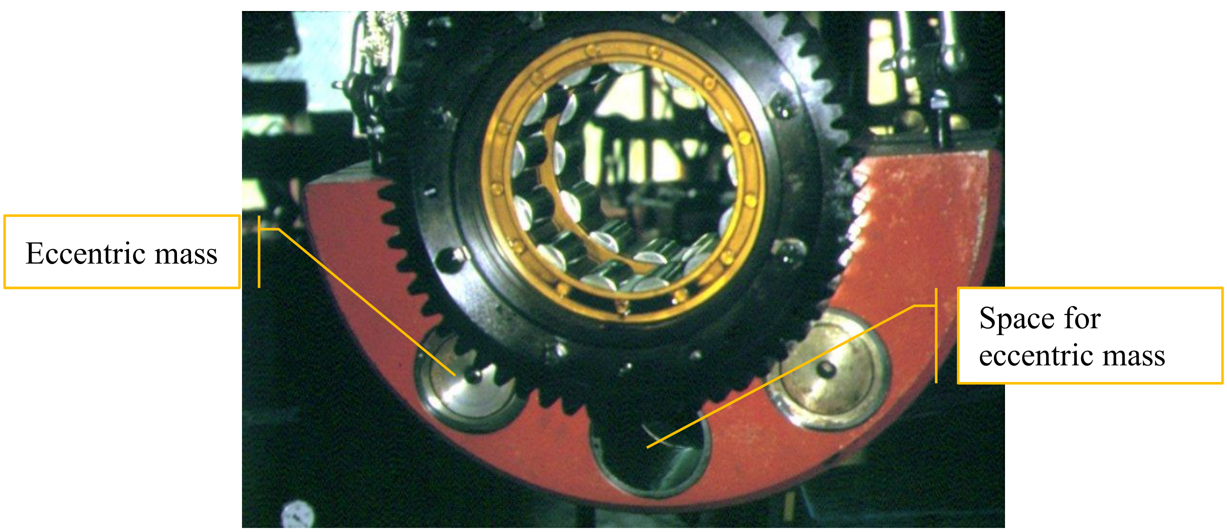

Fig. 1. Vibrator eccenter allowing for adding up to three additional masses.

As mentioned, the displacement amplitude given by vibrator manufacturers is usually in terms of double-amplitude and usually for a freely suspended vibrator (without clamp and attached pile/sheet pile). Typical performance characteristics of vibrators with stepwise variable eccentric moment are shown in Table 2.

TABLE 2. Müller Vibrators with stepwise variable eccentric moment (Müller-ThyssenKrupp 2004).

Vibrator	Units	MS-25 HHF	MS-50 HHF	MS-100 HHF	MS-120 HHF	M-200 HHF
Max. Centrifugal force	kN	750	1500	2500	3003	4000
Eccentric moment	kgm	25	50	100	116	190
Steps[2)]	kgm	12/15/ 20/25	24/30/ 40/50	48/60/ 80/100	80/94/ 110/116	(98)/110/ 150/190
Frequency steps	Hz	39.9/35.2/ 30.5/27.3	39.9/35.2/30.5/27.3	36/32/ 27.8/25	30.9/28.3/ 26.2/25.6	30/26/ 22.9
Static pulling force	kN	280	500	600	1200	1200
Dynamic mass[1)]	kg	2900	4500	7700	8900	11750
Total mass[1)]	kg	3700	6100	10900	15500	15500
Vibration (double) amplitude[1)]	mm	17.2	22.2	26.0	26.1	32.4
Vibration (double) amplitude[1)] (steps)	mm	8.3/10.3/ 13.8/17.2	10.7/13.3/17.8/22.2	12.5/15.6/20.8/26.0	18.0/21.1/ 24.7/26.1	16.7/18.7/ 25.5/32.4

[1)] without clamping device/pile; [2)]available eccentric moment

In the 1990s, vibrators with variable frequency and variable amplitude were introduced with resonance-free starting-up and shutting-down of vibratory driving. Such vibrators allow the operating frequency and eccentric moment (and thus amplitude) to be varied according to specific driving requirements and soil conditions. Typical performance parameters of vibrators with variable frequency and variable eccentric moment (amplitude) are shown Table 3. It should be mentioned that vibrator performance data of most modern manufacturers are similar to those shown in Table 1 through 3, for instance vibrators manufactured in North America (APE, ICE) or Europe (Dieseko, PTC, RTG).

2.3 Assessment of Vibrator Capacity

Vibrator manufacturers have provided empirical guidelines for the selection of vibrators. In Figure 2, the centrifugal force required for driving sheet piles is shown as a function of pile mass and depth of penetration.

TABLE 3. Resonance-free Müller Vibrators with variable frequency and eccentric moment (Müller-ThyssenKrupp 2004).

Vibrator	Units	MS-10 HFV	MS-16 HFV	MS-20 HFV	MS-28 HFV	MS-32 HFV	MS-48 HFV	MS-62 HFV
Centrifugal force	kN	610	968	1230	1473	1980	2960	2998
Eccentric moment (variable)	kgm	0-10	0-16	0-19.5	0-28	0-32	0-48	0-62
Speed	rpm	2358	2370	2400	2190	2375	2350	2100
Frequency	Hz	39.3	39.5	40.0	36.5	39.6	39.0	35.0
Static pulling force	kN	180	300	300	500	600	600	800
Dynamic mass w/o clamping device	kg	1700	2565	2530	3120	4850	6520	6850
Total mass[1)]	kg	2300	3530	3600	5320	7250	9700	11165
Vibration amplitude[1)]	mm	11.8	12.5	15.4	18.0	13.2	14.7	18.2

[1)] without clamping device/pile

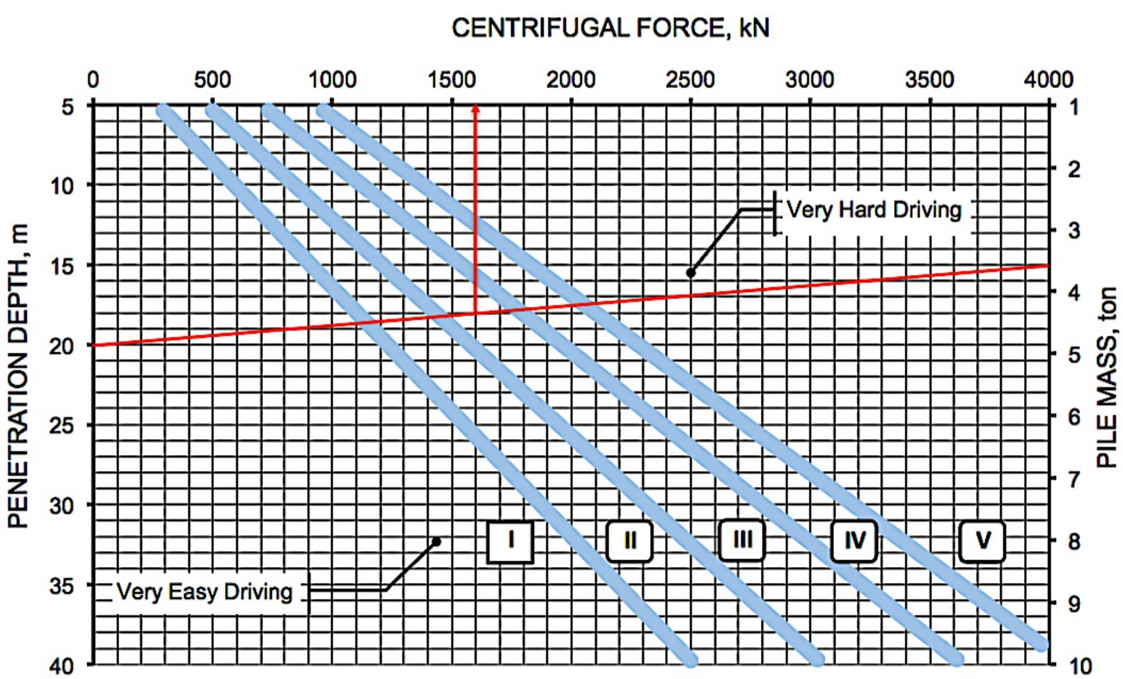

Fig. 2. Empirical chart for estimation of required vibrator centrifugal force for driving of sheet piles. The example (red lines): sheet pile mass: 3.6 ton maximum penetration depth: 20 m, driving into medium dense sand (III) requires vibrator with centrifugal force of at least 1,600 kN (based on Müller-ThyssenKrupp 2004).

The five soil categories depicted in Figure 2 are defined in Table 4.

Table 4. Drivability of sheet piles into granular soils, based on cone penetration tests, cf. Fig. 2.

Test Type	I Very loose	II Loose	III Medium Dense	IV Dense	V Very Dense
SPT, N (blows/0.3m)	< 4	4 - 10	10 - 30	30 - 35	> 50
CPT, q_c (MPa)	< 5	5 - 10	10 - 15	15 - 20	> 20
DPH[1)], N_{10} (blows/10 cm)	< 5	5 - 10	10 - 15	15 - 20	> 20

[1)] Heavy dynamic probing: mass 50 kg, height of fall 500 mm

Figure 2 and Table 4 are based on experience from sheet pile driving, compiled by one particular vibrator manufacturer (Müller-ThyssenKrupp, 2004). Such guidelines must be treated with caution. The guidelines assume that the user understands the limitations involved. In the example shown in Fig. 2, a pile with mass 3.6 ton shall be driven to a depth of 20 m into medium dense sand (III) (q_c ≈12 MPa). According to Figure 2, a vibrator with a centrifugal force of about 1,600 kN will be required.

An important aspect of Figure 2 is that the diagram was based primarily on experience from sheet pile projects, where the shaft resistance dominates. In the case of tubular piles, however, the toe resistance contributes significantly to the total driving resistance. Westerberg et al. (1995) suggested that the dynamic toe resistance during vibratory driving of piles is approximately equal to the CPT cone stress, q_c. Figure 3 shows the centrifugal force, required to drive a tubular pile with closed toe into granular soils of variable density, Massarsch and Fellenius (2017). The soil conditions are expressed according to the classification given in Table 4.

The application of Figure 3 is illustrated by the following example. The centrifugal force required to drive a tubular pile with a toe diameter of 350 mm into a granular soil with a cone resistance $q_c \approx 10$ MPa is approximately 950 kN. If the soil at the pile toe has a cone stress $q_c \approx 15$ MPa, the required centrifugal force will be approximately 1450 kN.

An additional parameter to be considered when assessing vibratory driving is the relative displacement of the pile to the soil. The relative displacement is particularly important for overcoming the shaft resistance in fine-grained (cohesive) soils. The larger the displacement amplitude, the more effective the driving process will be. The displacement amplitude depends on the total dynamic mass to be accelerated by the vibrator (dynamic mass) and the eccentric moment. In the following example, a sheet pile shall be driven by a vibrator (MS-28) with a 3,250-kg dynamic mass (vibrator and clamp). The mass of the 15 m long pile is 2,700 kg. The displacement amplitude can now be estimated from Eq. (2). The double displacement amplitude (2s) of the suspended pile, which is usually quoted by vibrator manufacturers, is 9 mm (M_e = 28 kgm). During sheet pile penetration, the vibration amplitude decreases, depending on the stiffness of the soil to be penetrated.

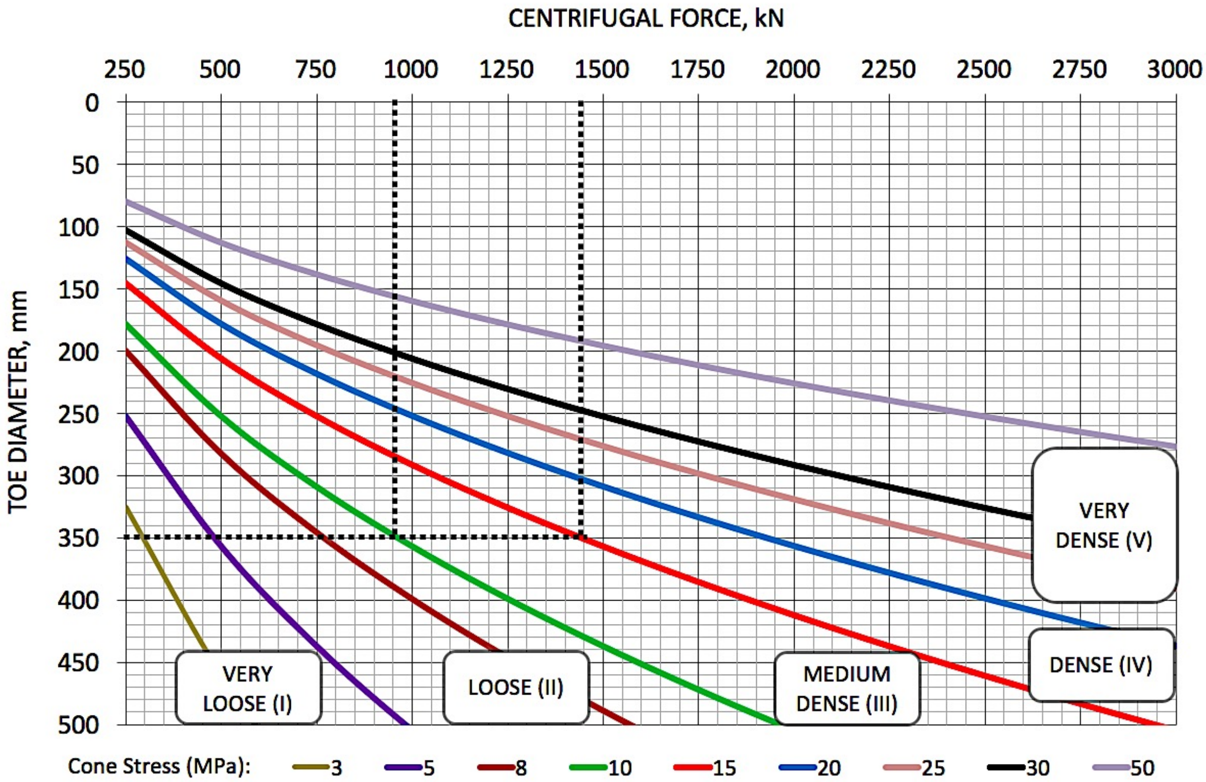

Fig. 3. Centrifugal force required to drive tubular pile (closed toe). Soil classification according to Table 4. The example illustrates the effect of cone stress on centrifugal force required to drive a pile with 350 mm toe diameter, Massarsch and Fellenius (2017).

3 MONITORING OF VIBRATORY DRIVING

3.1 Monitoring System

An important advantage of vibratory driving is that many aspects of the installation process can be monitored, controlled, and documented. With modern electronic measuring equipment, it is possible to acquire, display, and record information from different sensors, which can be mounted on the pile, the vibrator, the power pack, and the ground. Figure 4 shows the set-up of a vibratory monitoring system, which initially was developed for deep vibratory compaction.

When monitoring vibratory driving of piles and sheet piles, it is desirable that a data logger be used to record the following parameters:

- Position of pile/sheet pile
- Time of recording (at least one reading per second)
- Depth of sheet pile during penetration or extraction
- Operating frequency
- Acceleration of vibrator (pile top acceleration)
- Static force applied to the pile (pushing or lifting force)
- Hydraulic pressure of power supply
- Vibration velocity on or in ground (geophones).

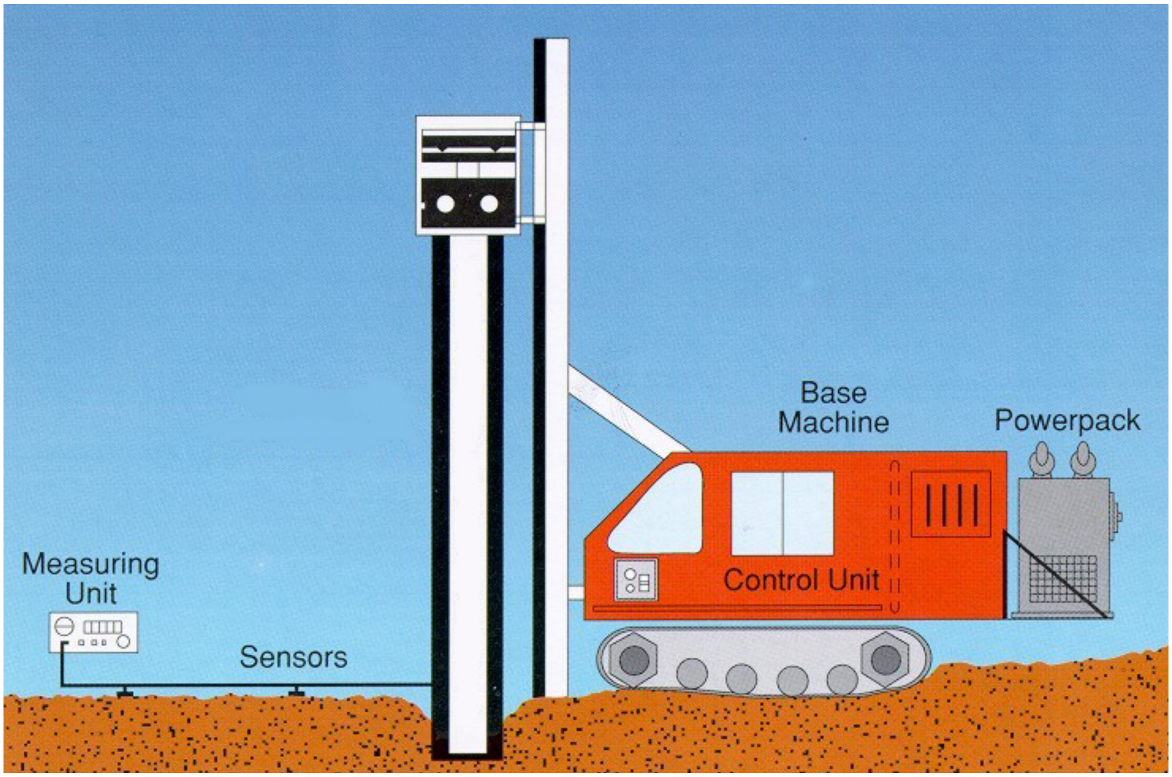

Fig. 4. Monitoring components of vibratory driving. Detail of control unit is shown in Fig. 5.

Ground vibrations can be recorded using geophones, Figure 5a, and can be displayed to the machine operator during vibratory driving, Figure 5b. Monitoring of information obtained during the vibratory driving process and response of the ground and/or of adjacent structures is an important aspect of modern vibratory works. For instance, in the case of vibratory driving in the vicinity of vibration-sensitive buildings or installations, a computer-operated system can be used to control the maximum vibration intensity in order to ensure that specified limiting values are not exceeded. Moreover, when vibrators are used for deep vibratory compaction, the vibration measurements can be used to guide the operation to ensure that maximum transfer of vibration energy is obtained from the vibrator/probe system to the surrounding soil, for example when using the resonance compaction system (Massarsch, 1991).

3.2 Optimization of Vibratory Driving and Compaction

From vibration monitoring, several important parameters can be derived such as: pile and soil displacement amplitude, resonance frequency of vibrator-pile-soil system, pile/probe penetration speed. These parameters can be displayed to the machine operator in real time and assist in the optimization of the driving or compaction system. The displacement amplitude provides important information regarding the dynamic mass to be accelerated by the vibrator. This information is particularly important when driving piles or sheet piles into cohesive soils. Soil compaction is enhanced when the vibrator is operated at the system resonance frequency, at which ground vibrations are strongly amplified. However, pile or sheet pile penetration will be low at the resonance frequency, at which the risk of vibration problems in surrounding buildings increases.

a) Ground vibration measurement by tri-axial geophone

b) Display of vibration parameters measured during pile penetration (System Gamperl und Hatlapa)

Fig. 5. Monitoring of ground vibrations and display of measurements to machine operator.

The pile penetration speed is another important parameter, which reflects the efficiency of vibratory driving. However, the penetration speed is relevant only when the pile or sheet pile is allowed to penetrate under the full weight of the vibrator.

3.3 Ground Vibrations during Resonance Compaction

The above described monitoring system has been used on a large number of projects, where resonance was used to increase the compaction efficiency in coarse-grained soil. In the following case history, a steel pipe pile (with closed toe) was vibrated into loose to medium dense sandy soil, using a Müller MS100 variable-frequency vibrator, see Table 2. The dynamic ground was monitored by a tri-axial geophone, placed 4 m away from the compaction point. The soil profile consisted of medium to dense sand, which prior to compaction was loose to medium dense. During the initial probe penetration phase, the compaction probe was vibrated at high frequency (35 to 39 Hz). During the compaction phase, the vibration frequency was gradually lowered until resonance and, thus, maximum vibration amplification was achieved (about 13 - 16 Hz). The response at the ground surface during vibratory driving is shown in Figure. 6. The vertical ground vibration velocity during penetration varied typically between 0 and 4 mm/s (average 2 mm/s), with peak values around 6 mm/s. At resonance (between 12 and 16 Hz), the vertical vibration velocity increased significantly, with maximum values reaching 20 mm/s. The vibration amplification was about 5 to 10. At high frequencies, the probe penetration velocity is usually significantly higher (by a factor of up to 10) compared to driving at resonance frequency.

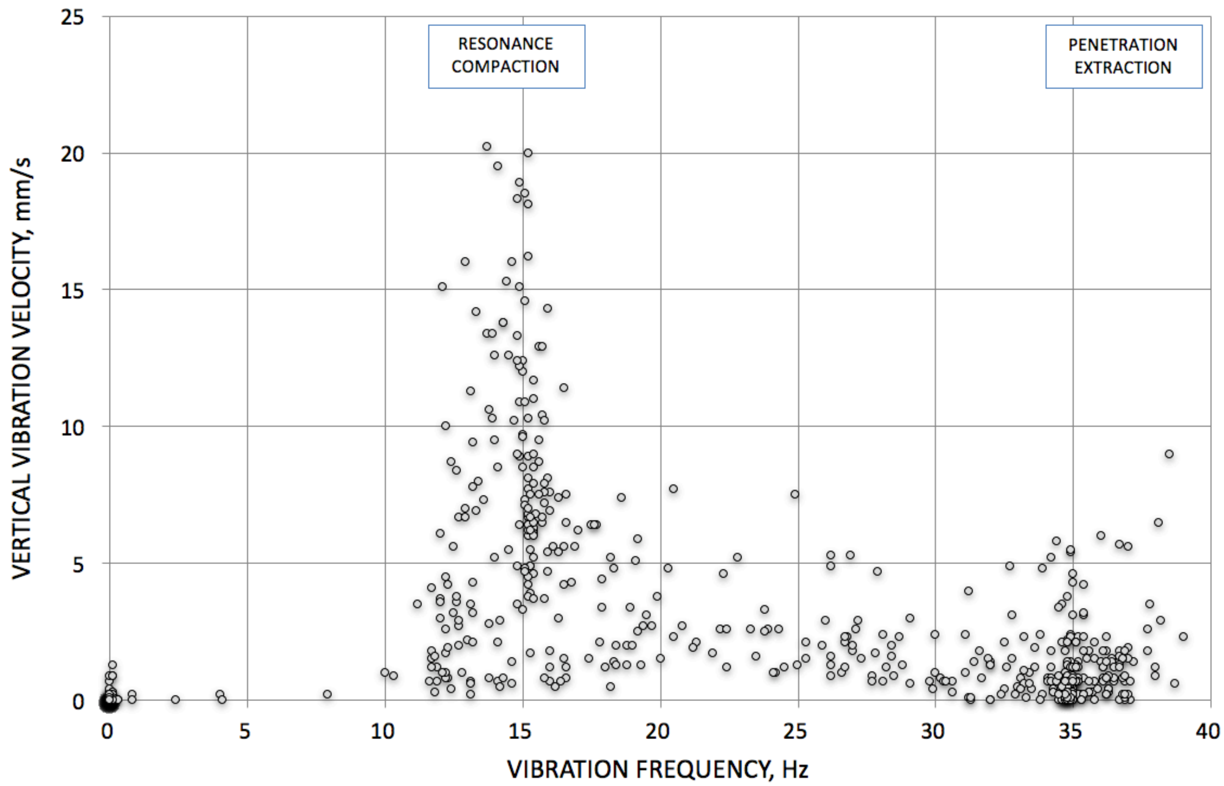

Fig. 6. Vertical vibration velocity measured at 4 m distance from the vertically oscillating compaction probe during penetration/extraction (32 – 38 Hz) and compaction phase (12 – 18 Hz).

Field monitoring of ground vibrations can provide valuable information regarding the transfer of vibration energy from the pile to the surrounding soil. It is apparent that efficient pile penetration occurs when the vibration frequency is significantly higher than the resonance frequency of the vibrator-pile-soil system. On the other hand, transfer of vibration energy and, therefore, the compaction effect is enhanced when the vibrator is operated at the resonance frequency.

4. ASSESSMENT OF VIBRATORY DRIVING PERFORMANCE

When a pile or sheet pile is to be installed by a vibrator, the selection of the driving equipment and the installation process should be carefully planned. The vibrator type and capacity as well as the driving parameters (operating frequency and amplitude) should be chosen, based on geotechnical information. Using an unsuitable vibrator will not only result in project delays and infer additional costs but under unfavorable conditions, also produce damaging ground vibrations in buildings or installations in the ground. The following paragraph proposes a concept which can be used to estimate vibrator centrifugal force, based on CPT results. It is recommended to determine the optimal vibratory driving process by field trials. The objective is to develop a correlation between CPT penetration resistance and pile penetration speed for a given vibrator type and pile size.

Rational design of a vibratory driving project requires site information that includes a well-established soil profile with soil description obtained from laboratory study of soil specimens. The best additional direct information is a continuous record of soil layering and density, such as provided by a CPTU sounding. The information obtained from SPT can also be valuable. In Europe, continuous light dynamic probing (DPL) or heavy dynamic probing (DPH) is frequently used for assessment of pile driving resistance. Unless information is available from vibratory driving from similar geology and equipment, a field trial is the best way of designing an installation of specific piles or sheet piles.

4.1 Predicting Driveability

The driveability of piles or sheet piles can be related to the penetration resistance from dynamic probing. In the case of heavy dynamic probing (DPH), a mass of 50 kg with a height of fall of 500 mm impacts on a steel rod and the number of blows for a penetration of 10 cm (N_{10}) is determined. An approximate relationship between different penetration testing methods and strength of granular soils is given in Table 4. It is possible to correlate the penetration resistance to the number of vibration cycles required for a pile or sheet pile to penetrate into granular soil. The speed of probe (pile or sheet pile) penetration can be measured (preferably recorded every second, but reported as penetration per minute). During driving, it is important that the vibrator rests fully on the probe and is not held back by the machine operator, as this will affect the probe penetration speed. It is also important, that the vibration frequency is high (above the resonance frequency) and kept as constant as possible. At a high vibration frequency, the vibration resistance will be caused primarily by the toe resistance of the probe, similar to the case at dynamic penetration testing.

In the following example, a sheet pile is driven by a vibrator operating at 40 Hz into a sand deposit consisting of several layers with variable DPL penetration resistance, blows/10 cm, shown in Figure 7.

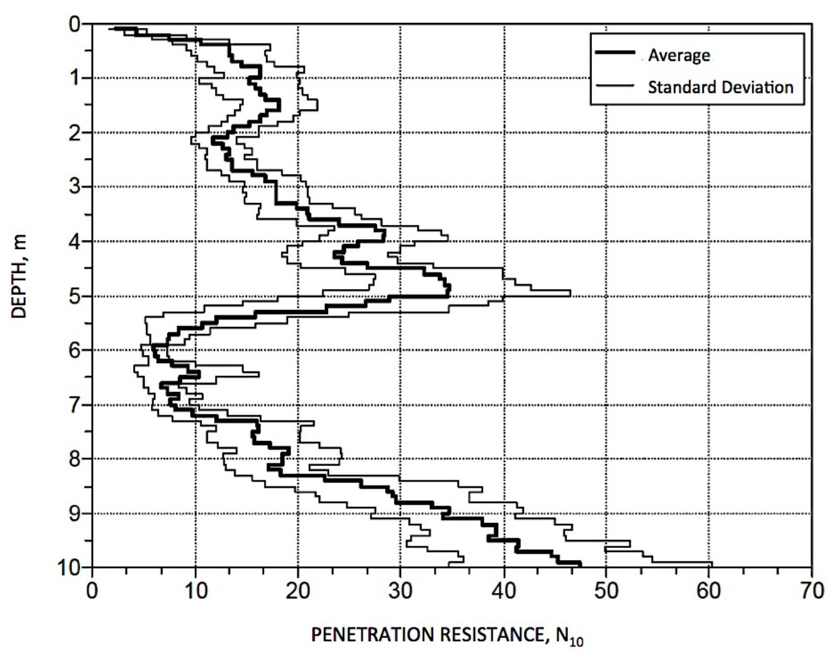

Fig. 7. Typical dynamic penetration test, Light Dynamic Penetrometer (DPL), from Schönit (2009). The standard for the DPL (10 kg, 500 mm height of fall) is to record the penetration as blows per 10 cm penetration).

From the depth measurement during vibratory driving of the sheet pile, the penetration speed can be determined as a function of depth. Figure 8 shows the penetration speed (m/s).

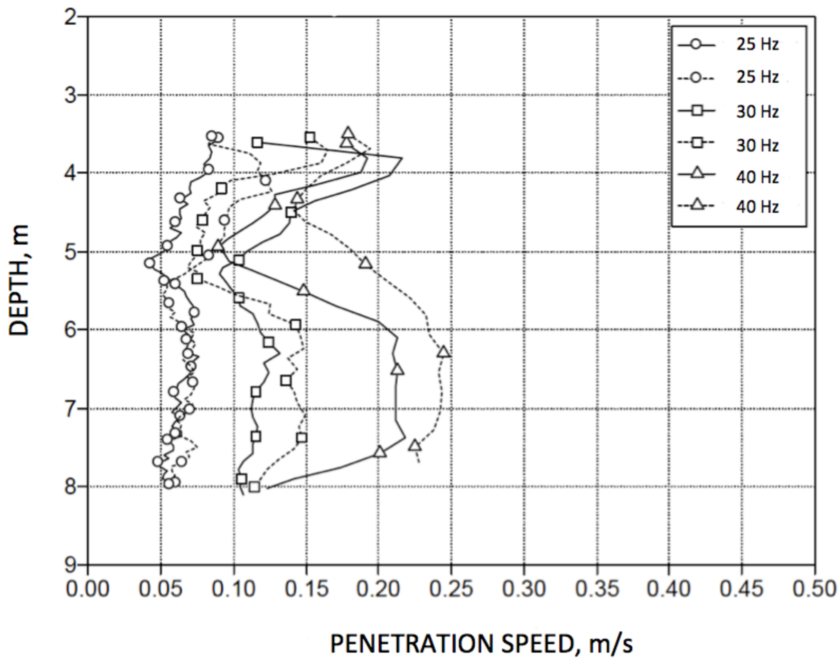

Fig. 8. Sheet pile penetration speed measured at three different vibration frequencies (two tests for each frequency), after Schönit (2009).

At field trials, the key vibrator performance parameters to record are the eccentric moment, the vibration frequency, the centrifugal force, and the displacement amplitude. It is important to record the displacement amplitude of the vibrator-pile system prior to driving of the pile penetration into the ground. The intensity of ground vibrations adjacent to the pile should also be recorded, as this information can be used to determine the risk of vibration amplification should the vibrator operating frequency be too close to the resonance frequency of the vibrator-pile-soil system. As the vibrator operating frequency, f (Hz or rpm) is known, it is possible to convert the measured pile penetration speed, v (cm/min) into an equivalent number of penetration cycles, c_e per depth interval (cycles/cm).

$$c_e = \frac{f}{v} \tag{3}$$

In the above example, the vibrator operating frequency was kept constant at 40 Hz (2,400 rpm). It is now possible to convert the penetration speed from Figure 8 into an equivalent number of vibration cycles, as shown in Figure 9. It is important to appreciate that the above shown correlation is soil-type specific and influenced by the type and capacity of vibrator, and the size and geometry (toe area) of the sheet pile.

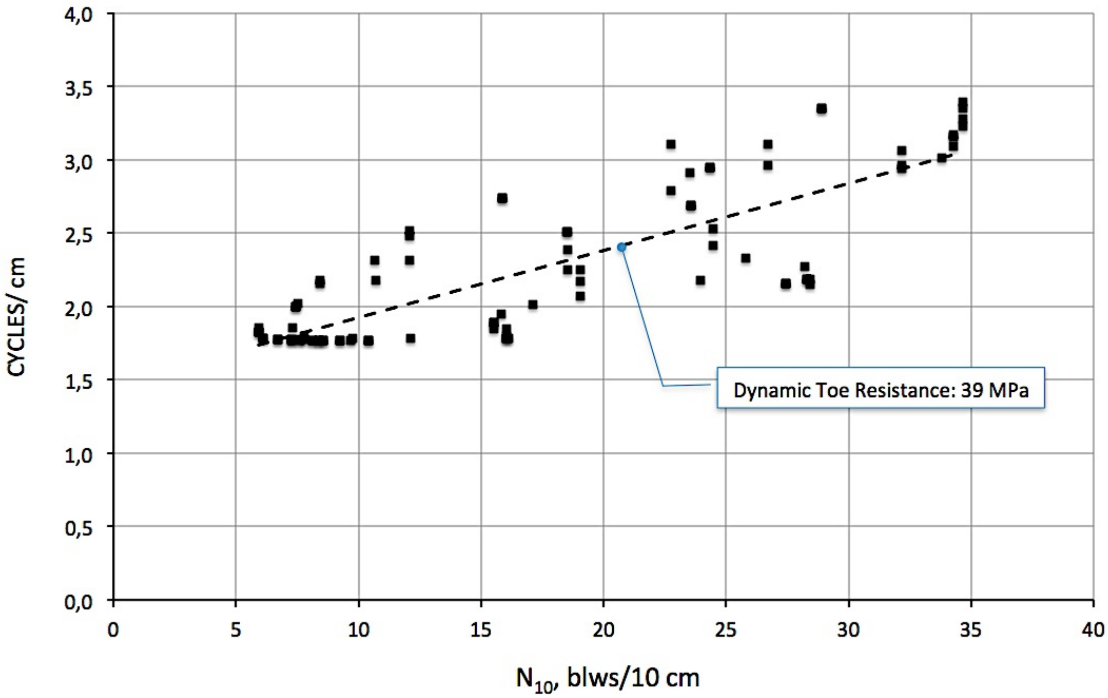

Fig. 9. Relationship between penetration resistance, N_{10} and number of vibration cycles at operating frequency of 40 Hz, cf. Figures 8.

5 ADDITIONAL APPLICATIONS OF VIBRATORY DRIVING

Vibrators can be used for the efficient installation of piles and sheet piles in coarse-grained soil. However, innovative solutions have been developed to broaden the range of vibrator applications and to facilitate vibratory driving in difficult soils.

At present, it is difficult to assess the bearing capacity of preformed piles, installed by vibratory driving. However, recent studies regarding the bearing capacity of cast-in situ piles have been published by Zeilinger and Hudelmaier (2009). They investigated the bearing capacity of cast-in-situ piles, installed by different driving methods (impact-driven, vibrated and vibro-jetted). Static and dynamic loading tests were performed on these piles that suggest that vibratory driving and water jetting does not have a detrimental effect on the static bearing capacity of in situ cast piles.

One interesting new concepts in foundation design during the past decades is the use of settlement-reducing elements (piled rafts). The foundation slab is supported by conical concrete elements, installed by vibratory driving (cf. Figure 11). This foundation concept is similar to ground improvement by stone columns or mixed in place columns. Prefabricated elements (concrete, steel or timber) can be used to increase the stiffness of loose or soft soil layers. However, vibrated conical elements do not suffer from the uncertainties associated with the installation of stone columns in soft, compressible soils. Massarsch et al. (1997) described a design concept, which is based on the load-sharing between the foundation slab foundation and reinforcing elements.

Fig. 11. Installation of conical concrete elements by vibrator with variable frequency.

6 CONCLUSIONS

Vibratory driving of piles and sheet piles is a common method, especially for the installation of foundation units and sheet piles. The most important vibrator parameters for vibratory driving are the centrifugal force, the vibration frequency, and thus also of the eccentric moment.

The vibrator-pile-soil interaction is a function of the resonance vibration frequency. Results of a theoretical analysis are presented which can be used to predict reliably the resonance of the vibrator-pile-soil system. The most important parameters governing the resonance frequency are the stiffness (shear wave speed) of the soil and the mass of the vibrator and the sheet pile. The eccentric moment does not affect the resonance frequency.

Empirical rules can be used to assess the required vibrator capacity (centrifugal force) for sheet pile driving in primarily coarse-grained soil. However, a more reliable concept can be used to predict the drivability of sheet piles or piles. A method is described which makes it possible to correlate the penetration resistance to the number of vibration cycles during sheet pile driving. A case history is presented which demonstrates the concept and shows that a correlation exists in coarse-grained soil between penetration resistance (dynamic penetrometer) and number of vibration cycles per depth interval. Based on this information, which can be obtained either from field trials or experience from past projects, it is possible to determine the penetration speed during vibratory driving. This information can be valuable when selecting the required vibratory driving equipment and predicting the duration of vibratory installation of piles or sheet piles.

Vibratory driving is very effective when installing conical ridged elements, which can be used to improve loose soil deposits.

7 ACKNOWLEDGEMENT

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