ABSTRACT: The limitations of energy-based vibration attenuation relationships are reviewed. The large scatter of reported results can be explained by the fact that such correlations do not take into account important dynamic pile and soil parameters. The propagation of driving energy from the hammer into the pile and from the pile to the soil along the shaft and at the base is discussed, using wave propagation theory. The importance of the impact velocity, the pile and soil impedance and of the distribution of the dynamic soil resistance are highlighted. The dynamic soil resistance, which depends on the strain-dependent impedance, is the source of ground vibrations. Guidance is given regarding the selection of soil impedance values, taking into account the strain-softening effect. Vibration propagation from the base and along the shaft can be calculated using analytical methods, which take into account the transfer of driving energy.

1 INTRODUCTION

Driven piles are among the most common and cost-effective foundation solutions in variable ground conditions. In recent years, equipment manufacturers have developed new, more powerful pile installation equipment, such as hydraulic impact hammers and sophisticated vibratory hammers. Electronic data acquisition systems have made it possible to monitor and document pile penetration and to determine the dynamic response of the ground and of buildings. With the introduction of wave equation computer programs, the prediction of the static pile capacity based on dynamic measurements during driving has become more reliable. Significant progress has also been made with respect to the determination of dynamic soil properties from seismic field and laboratory measurements.

Today, the efficient use of driven piles and sheet piles is many times limited due to concerns regarding negative environmental effects, such as excessive noise and ground vibrations or ground movements. In many cases, design engineers choose therefore alternative foundation solutions as they are not confident how to assess the risk of ground vibrations during pile driving.

At present, and in contrast to other aspects of pile dynamics, the prediction of ground vibrations is still based on crude empirical rules, developed about 30 years ago. While energy-based predictions of pile bearing capacity have been discarded due to their inaccuracy, these concepts are still being used to predict ground vibrations due to pile driving. These prediction methods neglect fundamental aspects of dynamic pile-soil interaction. Surprisingly few publications have addressed the problem of dynamic pile-soil interaction in a rational way (e.g. Martin, 1980; Massarsch, 1992; Selby, 1991). In the present paper, an attempt is made to describe in a rational way the parameters which govern the propagation of driving energy from the source to the surrounding soil layers, cf. Figure 1.
The following main aspects of vibration propagation during the driving of piles with an impact hammer can be identified: (A) – Wave propagation in the pile: the energy generated by the drop hammer (1) impacts the pile cap and the pile head (2) and the vibration energy is transmitted through the pile (3); (B) – Pile-soil interaction: along the pile shaft (4) and at the pile base (5); (C) – Wave propagation in the ground: transmission of vibrations through soil layers and the ground water; (D) – Dynamic soil structure interaction: dynamic response of buildings and installations in the building.

The dynamic soil-structure interaction of piled foundations at small strains is described extensively in the literature, such as the dynamic response of machine foundations (Novak & Janes, 1989). However, little information is available regarding the dynamic pile-soil interaction during driving at large strain levels (Massarsch, 1992).

Most investigations, which address vibrations caused by pile driving, discuss the propagation of vibrations in the ground and the response of buildings subjected to vibrations but do not consider the conditions at the source, i.e. the transfer of driving energy from the pile to the soil. Therefore, in this paper emphasis is placed on the discussion of parameters which govern the propagation of waves in the pile (A) and the dynamic pile-soil interaction (B), Figure 1.

### 1.1 Energy-based Vibration Attenuation

Most investigations reporting measurements of ground vibrations from pile driving have adopted concepts which were developed for the case of vibrations due to rock blasting. The vibration intensity at a distance from the source is assumed to be a function of the energy released at the source.

Attewell & Farmer (1973) analysed results of vibration measurements in different ground conditions and for different pile types. They came to the conclusion that the attenuation of ground vibration amplitude with distance from a pile is largely independent (!) of the geotechnical nature of the ground. They suggested that a conservative estimate of vibration velocity \( v \) at distance \( r \) from the energy source (pile) can be made from the following equation

\[
v = k \frac{\sqrt{E}}{r}
\]

where \( E \) is the input energy at the source and \( k \) is an empirical constant. It should be noted that the empirical factor \( k \) depends on the units used to define the distance and energy. Brenner & Viranuvut (1975) used equation (1) to compare results of vibration measurements from pile driving with information from projects reported in the literature, Figure 2.

Figure 2. Attenuation of peak particle velocity versus scaled energy (Brenner & Viranuvut, 1975).

In spite of the double-logarithmic scale, the scatter is large and would be unacceptable for most conventional geotechnical design applications. In spite of the uncertainties associated with this concept, the energy-based prediction of ground vibrations is still frequently used (New, 1986, Woods & Jedele, 1985, Jedele, 2005). One of the most widely referenced publications on this subject is the State-of-the-Art paper by Wiss (1981) on construction vibrations, which uses a similar relationship as the one proposed by Attewell & Farmer (1973). The peak particle velocity \( v \) is expressed as a function of the scaled distance

\[
v = K \left( \frac{D}{\sqrt{E}} \right)^{-n}
\]

where \( K \) and \( n \) are empirical constants and is \( D \) the distance from the energy source \( E \). However, guidance is not given in the literature how the distance should be chosen when a pile penetrates into the ground. This aspect will therefore be discussed in the following sections.

### 1.2 Influence of Pile Penetration Depth

The energy-base relationships require the assessment of the distance from the vibration source to the observation point. With the exception of the initial phase of driving when the pile penetrates dense soil layers close to the ground surface, the depth of the energy source will change as the pile penetrates into the ground. In most cases which have been reported...
in the literature, the horizontal distance at the ground surface is used when correlating results from vibration measurements. Figure 3 illustrates the problem for two measuring points, located at different distances from the pile. The distance to the energy source becomes particularly important in the case of measuring point A, which is located in the vicinity of a long pile. If the horizontal distance is used for all vibration measurements at location A during the penetration of the pile, it is not surprising that the scatter in reported values becomes large. On the other hand, if the distance to measuring point B corresponds to several pile length, the effect of pile penetration becomes less important. However, in most cases, problems associated with vibrations from pile driving occur at distances less than one pile length.

The location of the energy source during driving of a pile depends also on the soil resistance along the shaft and at the base. This case is illustrated in Figure 4, where two penetration resistance curves (e.g. cone penetration test) are shown. In the first case, it is assumed that the pile is driven into a sand deposit with gradually increasing stiffness. A significant amount of the driving resistance will be generated along the shaft of the pile. Waves will propagate mainly as cylindrical or conical shear waves, with wave attenuation similar to surface waves (Selby, 1991, Massarsch, 1992).

In the second case, it is assumed that a pile is driven into a clay layer with a dry crust (or surface fill) close to the ground surface. During the initial phase, vibrations will propagate along the ground surface in the form of surface waves. When the pile is driven into the soft clay layer, ground vibrations will be negligible. However, when the tip of the pile is driven into the dense bottom layer, most of the soil resistance will be generated at the base of the pile.

At the base of the pile, vibrations will propagate mainly in the form of body waves (compression and shear waves).

During impact driving, vibrations can be transmitted along the pile shaft and/or at the pile base. The source of vibration emission (origin of dynamic soil resistance) can change during pile penetration (shaft and/or base) and depends strongly on the geotechnical conditions. Thus, pile driving can give rise to vibrations emitted in the form of body waves at the pile base but at the same time also due to shear waves and surface waves along the shaft.

1.3 Influence of Pile Impedance

In addition to the geotechnical conditions, also the material properties of the pile are of importance. Heckman & Hagerty (1978) showed that the intensity of ground vibrations is affected by the dynamic properties of the pile material. In Figure 5 the $K$-value as defined in equation (2) is shown as a function of the pile impedance. From the reported measurements it can be concluded that ground vibrations increase markedly when the impedance of the pile decreases. Ground vibrations can be ten times larger in the case of a pile with low impedance compared to a pile with high impedance (Massarsch, 1992). This aspect will be discussed in more detail in the following sections.

2 VIBRATION PROPAGATION IN PILE

The propagation of the driving energy in a pile is a complex problem. During the past thirty years, major progress has been made in the area of pile dynamics. Today the use of dynamic testing methods is generally accepted and the assessment of pile capacity during driving is undertaken routinely.
The accuracy of dynamic methods for estimating the static pile capacity has been improved significantly. With the aid of modern measuring and data acquisition systems it is possible to monitor the dynamic force and wave propagation in the pile during driving. However, these concepts have not yet been used to analyse ground vibration problems associated with pile driving.

One of the main limitations of dynamic testing is that the total pile resistance, \( R_{tot} \), is composed of a static component, \( R_{stat} \), and a dynamic component, \( R_{dyn} \). As will be shown in the following sections, ground vibrations are caused by the dynamic (velocity-dependent) soil resistance. Therefore, it is important to determine \( R_{dyn} \) when predicting ground vibrations during pile driving. At first, the propagation of stress waves in the pile will be addressed. Thereafter, the dynamic interaction between the pile and the adjacent soil layers will be discussed.

### 2.1 Stress Waves in Piles

The vibration attenuation relationships given in equations (1) and (2) have major shortcomings as they are based on the assumption that energy in the pile driving system is conserved. The driving energy is determined from the potential energy of the hammer at the top of the stroke with reference to the pile top. A simplified model of a pile driving arrangement using an impact hammer with mass \( W \) and drop height \( h \) is shown in Figure 6.

The work that is done when the pile penetrates a distance \( s \) can be calculated if the assumption is made that the soil resistance acts along the pile shaft, \( R_M \) and at the pile point, \( R_P \). This aspect can be analysed using stress wave propagation theory, which can be used to study the pile penetration problem.

\[
F = Z v_p
\]  
where \( Z \) is the impedance of a pile with area \( A \). The pile impedance can be determined if the modulus of elasticity \( E \) is known. Alternatively, the impedance can be determined from the product of the cross-sectional area \( A \), the wave propagation velocity \( c \) and the material density of the pile \( \rho \)

\[
Z = \frac{AE}{c} = Ac \rho
\]

Inserting equation (4) into equation (3) yields the well-known relationship

\[
F = \frac{AE}{c} v_p
\]

which can be used to calculate the axial force in a pile based on measurement of the particle velocity during driving. When the hammer strikes the pile with the velocity \( v_0 \), a compression wave will be generated simultaneously in the pile and in the hammer. The hammer and the pile will remain in contact only for a short time, the impact time. Since the force between the hammer and the pile must be equal

\[
Z_H v_H = Z_P v_P
\]
where $Z_H$ and $Z_P$ are the impedances of the hammer and of the pile, respectively. The corresponding particle velocities are $v_P$ and $v_H$, respectively. Since the particle velocities in the hammer and in the pile are the same at the contact surface

$$v_0 - v_H = v_P \quad (7)$$

Combining equations (6) and (7) and rearranging the terms, the particle velocity $v_P$ in the pile can be calculated from

$$v_P = \frac{v_0}{1 + \frac{Z_P}{Z_H}} \quad (8)$$

In the case that $Z_H = Z_P = Z$ the vibration velocity in the pile $v_P$ will be half the hammer impact velocity

$$v_P = 0.5v_0 \quad (9)$$

Thus the particle velocity in the pile will correspond to half the initial velocity $v_0$ of the impacting hammer. The force $F_i$ during the impact depends thus on the striking velocity of the hammer $v_0$ and on the impedance of the pile $Z_P$ and can be calculated from

$$F_i = 0.5v_0Z_P \quad (10)$$

Immediately after the hammer strikes the pile the particle velocity in the pile behind the wave front will be $v_0/2$, cf. Figure 6b. The duration of the impact, when the pile and the hammer are in contact, $t_0$ will be equal to

$$t_0 = \frac{2L_H}{c} \quad (11)$$

where $L_H$ is the length of the hammer. The length of the stress wave in the pile will thus be $2L_H$, cf. Figure 6c. If the material properties in the hammer are different to that in the pile but the impedances are the same due to difference in cross section, the duration of the impact will then be

$$t_0 = \frac{2L_H}{c_H} \quad (12)$$

where $c_H$ is the velocity of the stress wave in the hammer. The length of the stress wave in the pile will in this case be

$$L = 2L_H\frac{c_P}{c_H} \quad (13)$$

If an infinitely rigid hammer impacts an elastic pile, the top of the pile will be set in motion at the velocity of the impacting hammer. The force generated in the pile slows down the motion of the hammer and a stress wave is generated in the pile (Goble, 1995). In the next instant the hammer will be moving slower and the generated particle velocity will be smaller. The force at the top of the pile $F_i$ will decay exponentially according to the relationship

$$F = F_ie^{-\frac{Z_P}{M_H}} \quad (14)$$

where $M_H$ is the mass of the hammer and $F_i$ is the force at impact given by equation (5). With some simple algebraic modifications equation (14) can be modified to the form

$$F = F_ie^{\frac{M_P}{M_H}} \quad (15)$$

where $M_P$ is the mass of the pile and $\alpha$ is a variable expressing the time in $L/c$ units. It is thus apparent that the force in the pile is also affected by the ratio of the pile and hammer mass.

2.1.1 Maximum Force at Pile Base

When the initial wave $F_i(t)$ reaches the pile base it starts to move. The force $F_p(t)$ at the pile base will increase with increasing displacement. At equilibrium

$$F_p(t) = F_i(t) + F_i(t) \quad (16)$$

where $F_i(t)$ is the reflected wave. For the case that the material below the pile is infinitely rigid, which is of interest in the case of pile vibrations, then $F_i = F_i$ and $F_p = 2F_i$. In this case, the downward directed compression wave will be reflected and the stress will increase by up to 100%. Thus, it is possible to estimate with rather simple theory the upper limit of the force $F_p$ that can occur at the pile base during hard driving.

2.1.2 Impact Velocity vs. Energy

To illustrate the limitation of the energy concept, two pile driving cases are compared. It is assumed that both piles are driven with the same energy. At first a hammer with a mass of 4 tons strikes the pile from a height of 1 m, yielding an energy of 40 kJ. The impact velocity $v$ is obtained from

$$v = \sqrt{2gh} \quad (17)$$

It should be pointed out that the impact velocity is independent of the mass of the hammer. At a drop height of 1 m, the velocity at hammer impact is 4.3 m/s. In the second case, a hammer with a mass of 2 tons strikes the pile from a height of 2 m, thus generating the same energy (40 kJ). However, the impact velocity is 6.3 m/s. The stress in the pile $\sigma_P$ can be calculated, based on equation (5), from the following relationship
The stresses in the respective piles are calculated, using the material properties given in Table 1. In the case of a concrete pile and assuming the same driving energy, but increasing the drop heights (from 1 to 2 m), the stress in the pile increases from 44 MPa to 63 MPa.

Table 1. Typical Material Properties for Driven Piles

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$ (kN/m$^3$)</th>
<th>Modulus $E_p$ (MPa)</th>
<th>Wave velocity, $C_p$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>78.5</td>
<td>210 000</td>
<td>5120</td>
</tr>
<tr>
<td>Concrete</td>
<td>24.5</td>
<td>40 000</td>
<td>4000</td>
</tr>
<tr>
<td>Timber</td>
<td>10</td>
<td>16 000</td>
<td>3300</td>
</tr>
</tbody>
</table>

2.1.3 Maximum Stress in Pile

It may be of interest to determine maximum force that can be propagated in the pile. The maximum stress in a pile can be calculated by inserting equation (17) in equation (18)

$$\sigma_p = \frac{E_p}{c_p} \sqrt{2gh_c}$$

from which the critical drop height $h_c$ can be calculated for different pile materials

$$h_{crit} = \frac{\sigma^2_{max}}{2g\rho E}$$

In the case of a concrete pile with cylinder strength ranging between 30 and 60 MPa and the material properties listed in Table 1, the critical (effective) drop height varies between 0.47 m and 1.9 m. In the case of a steel pile with a dynamic strength of 450 MPa, the critical drop height increases to 6.3 m.

The above brief discussion demonstrates that stress wave propagation during pile driving is affected by several factors, such as hammer weight, hammer impact velocity and pile impedance. It is thus not surprising that a single parameter, driving energy, cannot describe the pile driving operation correctly.

2.2 Pile-Soil Interaction

The total soil resistance $R_{tot}$ during pile driving is composed of a displacement-dependent (static) component $R_{stat}$ and a velocity-dependent (dynamic) component $R_{dyn}$

$$R_{tot} = R_{stat} + R_{dyn}$$

2.2.1 Dynamic Base Resistance

The soil resistance below the pile can be modelled conceptually as a spring with stiffness $k$ and a dashpot with viscous damping $c$ ($J_c Z_p$) Figure 7. When the pile base is moved a distance $u$, the total force at the pile base, $R_{tot}$ is resisted by a static component, which depends on spring stiffness $k$, and on a dynamic component, which depends on damping, $c$ ($J_c Z_p$).

Goble et al. (1980) suggested that the dynamic resistance at the tip of the pile can be expressed by the following relationship

$$R_{dyn} = J_c Z_p v_p$$

where $J_c$ is a dimensionless damping factor. It is generally assumed that $J_c$ depends only on the dynamic soil properties. Typical values of $J_c$ were determined empirically and typical values are given in Table 2.

Table 2. Damping factor $J_c$ for different soils (Rausche et al. 1985).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$J_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>0.60 – 1.10</td>
</tr>
<tr>
<td>Silty clay and clayey silt</td>
<td>0.40 – 0.70</td>
</tr>
<tr>
<td>Silt</td>
<td>0.20 – 0.45</td>
</tr>
<tr>
<td>Silty sand and sandy silt</td>
<td>0.15 – 0.30</td>
</tr>
<tr>
<td>Sand</td>
<td>0.05 – 0.20</td>
</tr>
</tbody>
</table>

A fundamental aspect of predicting ground vibrations due to pile driving is that ground vibrations are caused by the dynamic soil resistance. Thus, if a pile is pushed slowly into the ground, the dynamic resistance does not exist and ground vibrations will be negligible. If the penetration velocity increases, the dynamic soil resistance ($R_{dyn}$) increases, and consequently giving rise to ground vibrations. Thus, the dynamic soil resistance and ground vibrations are closely associated, as will be discussed in the following sections.
The damping factor \( J_e \) is generally considered to depend only on the dynamic soil properties. However, Iwanowsky & Bodare (1988) derived the damping factor \( J_e \) analytically, using the model of a vibrating circular plate in an infinite elastic body. In this way it was possible to describe the interaction between the pile base and the surrounding soil. They showed that the damping factor \( J_e \) depends not only on the soil but also on the impedance of the pile at the tip. They arrived at the following relationship

\[
J_e = \frac{Z_s}{Z_p}
\]

(23)

which implies that the damping factor is a function of the ratio of the soil impedance and the pile impedance. The dynamic component of the driving resistance at the tip can thus be readily calculated from

\[
R_{dyn} = 2Z_s v_p
\]

(24)

The damping factor \( J_e \) does not appear in equation (24), cf. equation (22). Instead, the soil impedance \( Z_s \) is sufficient to determine the dynamic behaviour at the pile-soil interface. In Table 3, typical \( J_e \) damping values are calculated according to equation (23) for pile with material properties, as stated in Table 1. Thus, it is possible to determine \( J_e \) factors for different pile types, geometries and material properties.

Table 3. Values of the \( J_e \) damping factor for different pile materials. An average soil density of \( \rho = 1.8 \) t/m\(^3\) was chosen. For pile material properties cf. Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Compression wave velocity at pile base, ( c_{p,m} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.02 0.04 0.07 0.09 0.11 0.13</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.09 0.18 0.28 0.37 0.46 0.55</td>
</tr>
<tr>
<td>Wood</td>
<td>0.27 0.55 0.82 1.09 1.36 1.64</td>
</tr>
</tbody>
</table>

2.3 Dynamic Soil Resistance at Base of Pile

The dynamic soil resistance below the pile base can be modelled using the theoretical concept developed by Herlitz (1984). He presented a closed-form solution of the displacements of the centre of a circular plate for arbitrary time-dependent forces acting in an elastic body. He also gave the expression of the displacement for a step-load, which was used by Bodare & Orrje (1985) to describe the response of a plate subjected to a quadratic sinusoidal force. The displacement \( u_0 \) at the centre of a circular plate inside an elastic medium, when subjected to a uniformly distributed step load \( F_0 \) can be described by the following three terms

\[
u_0 = F_0 \frac{s}{2\pi a^2 z} t \quad 0 \leq t \leq t_p
\]

\[
u_0 = F_0 \frac{1}{3m} \left( t^2 + t_p^2 \right) \quad t_p \leq t \leq t_s
\]

\[
u_0 = F_0 \frac{1 + s^2}{4\pi Ga} \quad t_s \leq t
\]

(25)

using the following definitions

\[
s = \frac{c_s}{c_p} = \frac{1 - 2\nu}{\sqrt{2 - 2\nu}}
\]

(26)

where \( \nu \) is Poisson’s ratio. The specific impedance of the material \( z \) is defined as

\[
z = c_s \rho = \sqrt{G \rho}
\]

(27)

where \( \rho \) is the material density and \( G \) is the shear modulus. The mass of the sphere with a radius \( a \) in the elastic material is defined as follows

\[
m = \frac{4}{3} \pi a^3 \rho
\]

(28)

The parameters \( t_s \) and \( t_p \) are the times for a transverse or longitudinal wave to travel one radius \( a \)

\[
t_s = \frac{a}{c_s} \quad t_p = \frac{a}{c_p}
\]

(29)

By differentiation of the first part of equation (25) with respect to time, it can be seen that the velocity at the centre of the plate is constant

\[
v_0 = F_0 \frac{s}{2\pi a^2 z}
\]

(30)

In the second part, the acceleration \( a_0 \) at the centre is constant

\[
a_0 = F_0 \frac{2}{3m}
\]

(31)

In the third part the displacement is constant and is the same as the static displacement

\[
u_0 = F_0 \frac{1 + s^2}{4\pi Ga}
\]

(32)

The displacement \( u_0 \) at the centre and the average displacement \( u_{av} \) is illustrated in Figure 8.

For the case of a plate inside an elastic medium, the relationship between force \( F_0 \) and the particle velocity \( v_0 \) is given by equation (30). This relationship can be rewritten to express the stress as a function of the velocity

\[
\sigma_0 = \frac{2z}{s} v_0 = 2\rho c_p v_0
\]

(33)
This stress-particle velocity relationship according to equation (33) is shown in Figure 9. Orrje (1996) used this relationship to analyse the dynamic response of a plate impacting on an elastic half space.

Figure 9 demonstrates that the specific impedance $z$ can be determined from field tests if the mobilized dynamic stress is plotted versus the velocity during impact of the plate. By rearranging the terms of equation (30) it is possible to determine the particle velocity that is transmitted from the plate to the underlying soil. The vibration velocity $v_{\text{max}}$ which is generated due to the mobilized stress, $\sigma_{\text{mob}}$, can be determined from the following equation

$$v_{\text{max}} = \frac{\sigma_{\text{mob}}}{\rho c_s^*} s$$

(34)

where $c_s^*$ is the shear strain-dependent shear wave velocity.

The shear wave velocity decreases with strain level and can be calculated from

$$c_s^* = R_c c_s$$

(35)

where $c_s$ is the shear wave velocity at small strain and $R_c$ is the wave velocity reduction factor. The effect of shear strain on the deformation properties of fine-grained soils was discussed in detail by Massarsch (2004). Semi-empirical solutions are available to estimate the shear wave velocity at small strain and the effect of shear strains. Massarsch (2004) showed that in the case of fine-grained soils the shear wave reduction factor, $R_c$, depends on the plasticity index, $I_p$, of the soil. In Figure 10, the influence of shear strain on the shear wave reduction factor $R_c$ is given.

In soils with low plasticity (sandy soils), the shear wave velocity at large strain (failure) can decrease by over 70%. For example, in a sandy silt, the shear wave velocity can decrease from its maximum value of around 100 to 150 m/s at small strains ($<$10^{-3} %) to about 30 to 50 m/s at failure (strains exceeding 1 %). In soils with higher plasticity, the reduction of the shear wave velocity will be considerably smaller. In case of a plastic clay ($I_p = 50\%$) the shear wave velocity will decrease by about 45 % of its maximum value. The effect of shear strain on wave propagation velocity and thus also on the soil impedance is generally neglected, but is of great importance for accurate predictions of pile-soil interaction and wave propagation in the near-field.

2.4 Dynamic Soil Resistance along Shaft

The dynamic force transmitted to the soil $F_{Dm}$ can be calculated based on the following relationship

$$F_{Dm} = v_{\text{max}} z^* = v_{\text{max}} \rho c_s^*$$

(36)

Figure 8. Schematic illustration of the displacement at the centre, $u_0$ and the average displacement $u_{av}$ due to a step load uniformly distributed across a circular area of radius $a$, Bodare & Orrje (1985), cf. equation (25).

Figure 9. Relationship between dynamic stress $s$ and velocity $v$ during dynamic loading of a plate in an elastic material, cf. equation (32).

Figure 10. Reduction of shear wave velocity as a function of Plasticity Index, $I_p$, for different shear strain levels, cf. equation (35).
where $v_{\text{max}}$ is the maximum velocity of the pile and $z^*$ is the strain-dependent impedance of the soil at the interface with the pile. As mentioned previously, the shear wave velocity – and thus the impedance of the soil - decreases with strain level, cf. Figure 10. Thus, there is an upper limit to the vibration energy which can be transmitted along the pile shaft (and at the pile base). The maximum vibration velocity, $v_{\text{max}}$ which is transmitted from the pile shaft to the soil can be estimated from

$$v_{\text{max}} = \frac{F_{\text{Dm}}}{\rho c_s}$$

(37)

3 PREDICTION OF GROUND VIBRATIONS

3.1 Dynamic Pile-Soil Interaction

In order to make rational predictions of ground vibrations during pile driving, it is important to consider the most important factors, which control the transfer of driving energy. As has been shown above, these factors are: hammer impact velocity and mass, pile impedance and dynamic soil resistance along the shaft and at the base of the pile. It is possible to estimate – based on the above given relationships - the upper limits of the vibration velocity which can be transmitted along the pile shaft and at the base.

The significance of different parameters can be assessed by relatively simple analytical methods. However, a more reliable prediction of vibration propagation in the pile can be made based on dynamic measurements during pile driving. Stress wave measurements have gained widespread acceptance and details of performing such measurements are not presented in this paper. Figure 11 shows a principle drawing of the results of stress wave measurements in a pile driving. From the velocity and strain measurements, the force in the pile and the shaft resistance can be readily determined.

Based on such measurements it is possible to obtain a rather clear picture of the location (source) and extent of vibration emission from the pile shaft and at the base. It is also possible to obtain the input parameters (mobilized resistance) for assessing the dynamic interaction between the pile and the soil along the shaft and at the base.

3.2 Vibration Propagation in Elastic Materials

Sophisticated computational methods are available for studying vibration propagation in complex ground conditions. However, the propagation of vibrations in the soil can then be assessed using relatively simple, well-established methods (Massarsch, 1992).

The vibration amplitude $A_2$ at distance $R_2$ can be calculated if the vibration amplitude $A_1$ is known at distance $R_1$

$$\frac{A_2}{A_1} = \left(\frac{R_2}{R_1}\right)^{-n} e^{-\alpha (R_2 - R_1)}$$

(38)

The exponent $n$ depends on the wave type as shown in Table 3.

<table>
<thead>
<tr>
<th>Wave type</th>
<th>Exponent n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Wave</td>
<td>1.0</td>
</tr>
<tr>
<td>Body Wave at surface</td>
<td>2.0</td>
</tr>
<tr>
<td>Surface wave</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The attenuation of ground vibrations is strongly affected by the absorption coefficient $\alpha$, which depends on the soil damping coefficient $D$, the vibration frequency $f$ and the shear wave velocity $c_s$ (for the case of compression waves, $c_p$ should be used)

$$\alpha = \frac{2\pi D f}{c_s}$$

(39)

For the case of vibration propagation in an elastic medium (far-field problem), the soil damping coefficient $D$ is on the order of 3 – 6%. However, at large strain in the near field of the vibration source, soil damping can increase significantly.

Values of the absorption coefficient $\alpha$, given in the literature vary within a wide range and make a rational analysis difficult. Comparison of measured and predicted vibration velocity values based on equation (38) and (39) show good agreement. However, it is important that the effect of shear strain level on the wave velocity is taken into consideration.
CONCLUSIONS

Empirical vibration attenuation relationships, which are based on the energy transmitted to the pile using the concept of scaled distance, are not reliable, as they do not take into account basic pile dynamic considerations. Two major shortcomings are the definition of the scaled distance using hammer impact energy and the fact that dynamic pile-soil interaction is neglected. Thus, it is not surprising that the scatter of reported results is large and not acceptable for reliable prediction of ground vibrations.

The fundamental aspects of hammer-pile-soil interaction are presented, which make it possible to estimate the particle velocity and thus the dynamic forces in the pile. Instead of the hammer energy, the impact velocity at the pile head is a more suitable parameter on which to base prediction of ground vibrations.

The distribution of the dynamic soil resistance along the pile shaft and at the pile base depends on the dynamic pile and soil properties. A fundamental parameter required for the analysis of pile vibration problems is the impedance. The main advantage of stress wave theory is that the relative importance of different parameter can be evaluated in a rational way, thereby giving a better understanding of the complex problem.

The most important aspect of dynamic pile-soil interaction is the fact that the intensity of ground vibrations depends on the impedance of the soil and on the particle velocity. It is possible to estimate the dynamic response based on theoretical considerations. This information can be used to select appropriate driving equipment and pile types.

The most reliable method of predicting ground vibrations is by stress wave measurements during pile driving. In this way it is possible to determine the source of vibration during different phases of pile installation.

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