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Fundamentals of the vibratory driving of piles and sheet piles

Vibratory driving is a common method for installing or extracting piles and sheet piles as well as for deep vibratory compaction. The most important parameters are vibration frequency, vibration amplitude and eccentric moment. These parameters govern vibratory driving and, in particular, the soil resistance at the toe and along the shaft of a pile. Pile driving causes oscillating horizontal ground vibrations in coarse-grained soils. It can be shown that these horizontal vibrations reduce the shaft resistance during driving. The process results in a permanent increase in the horizontal effective stress, which causes arching around the vibrated pile. The increase in horizontal effective stress is also important for deep vibratory compaction. The resonance frequency of the vibrator-pile-soil system significantly affects pile penetration and the emission of ground vibrations. At resonance, the vertical vibration velocity in the soil reaches a maximum and pile penetration becomes very slow, whereas beyond resonance, the vibration velocity decreases and the pile penetration speed is high. Field monitoring of the vibratory driving process can be used to optimize vibratory pile driving, as examples have shown. A new concept is proposed in which the driveability can be determined from a correlation between penetration resistance measurements (blows/depth) and penetration speed. The validity of the concept is demonstrated by a case history and project references.

Grundlagen des Vibrationsrammens von Pfählen und Spundbohlen. *Vibrationsrammen ist ein effektives Verfahren zum Einbringen oder Ziehen von Spundwänden und Pfählen sowie für die Tiefenverdichtung. Die wichtigsten Parameter sind Schwingfrequenz, Schwingamplitude und statisches Moment. Diese Parameter bestimmenden den Vibrationsvorgang und, insbesondere, den Bodenwiderstand an der Pfahlspitze sowie längs des Pfahlmantels. Das Vibrationsrammen verursacht horizontale Bodenschwingungen in grobkörnigen Böden die den Bodenwiderstand während des Ramm- oder Ziehvorganges reduzieren. Durch das Vibrationsrammen wird außerdem die horizontale Effektivspannung dauerhaft erhöht und verursacht eine Gewölbebildung um den Pfahl. Die Erhöhung der horizontalen Effektivspannung ist auch wichtig bei der Tiefenverdichtung mittels Vibratoren. Die Resonanzfrequenz des Vibrator-Pfahl-Boden Systems ist ein wichtiger Faktor der den Eindringvorgang von Pfählen und Spundbohlen, sowie die Ausbreitung von Bodenschwingungen beeinflusst. Bei Resonanz erreicht die vertikale Schwingungsgeschwindigkeit im Boden ein Maximum und das Eindringen des Pfahls in den Boden verlangsamt sich markant. Beim Einvibrieren oder Ziehen von Pfählen deutlich über der Resonanzfrequenz nehmen die Bodenschwingungen ab und die Eindringgeschwindigkeit von Pfählen oder Spundbohlen erhöht sich. Durch das Messen verschiedener Schwingparameter kann der Rammvorgang optimiert werden. Ein neues Konzept zur Beurteilung der Rammbarkeit von Pfählen wird vorgestellt, das von der Beziehung zwischen Sondierwiderstand und Eindringgeschwindigkeit des Pfahls ausgeht.*

1 Introduction

Vibrators play an important role in many construction processes and, in particular, are used for the installation and/or extraction of piles and sheet piles. In many countries, and in Europe especially, new innovative vibratory driving equipment – hydraulic vibrators in particular – has been developed, such as vibrators with variable operating frequency and variable amplitude (eccentric moment). Vibrators can either be suspended from a crane or guided by leads depending on the specific application. The operation of modern hydraulic vibrators can be controlled, monitored and documented during all phases of operation.

The geotechnical literature describes many successful applications of vibrators for driving piles or sheet piles or installing casings. Vibratory driving can often achieve high production rates with minimal environmental impact. However, vibratory driving should be chosen with careful consideration for the project-specific conditions, such as the geotechnical and hydrogeological situation, environmental considerations, etc. Otherwise, environmental effects (such as ground vibrations) can have negative consequences. The selection of vibrators for specific applications is frequently based on empirical rules. Unfortunately, many foundation companies leave the choice of vibrator type and the selection of operating parameters to mechanical engineers, who often have insufficient understanding of fundamental geotechnical principles. When using vibrators to install piles, the following three aspects need to be addressed:

- **Driveability**: Select appropriate vibrator system and operation to assure efficient performance.
- **Bearing capacity**: Verify that the required bearing capacity can be achieved.
- **Environmental impact**: Minimize ground vibrations and/or noise.

The objective of this paper is to show that the three aspects mentioned are closely linked and one aspect cannot be addressed without considering the others. The paper focuses on the analysis of the pile/sheet pile driving process and on how to optimize the installation process by varying the machine parameters during installation or extraction. Consideration is also given to the important, but often neglected, aspect that ground vibrations generated during vibratory driving in coarse-grained soils also cause a permanent increase in horizontal stress and induce preconsolidation. Such stress changes affect the static bearing capacity of piles as well as the compaction effect in sands. Although of great practical importance, a detailed discussion of the environmental aspects of vibratory driving (ground vibrations and permanent soil displacement) are beyond the scope of this paper. Static soil displacements due to the installation of piles have been described in [1]. Vibrations caused by pile driving have been discussed in [2].

2 Vibratory driving

Modern vibrators can be controlled and adapted to achieve optimal performance. In the following, the term pile will be used but the concepts apply also to the driving of sheet piles, casings or compaction probes. An important advantage of modern vibrators is that machine parameters can be varied to achieve efficient driving or compaction. Vibratory excitation affects a pile in a different way to impact driving. In the case of vibratory driving, the pile is rigidly connected to the vibrator, resulting in minimal energy loss as energy is transferred from vibrator to pile. The vibration frequency is relatively low, typically below 40 Hz (2400 rpm). Thus, the wavelength propagating down the pile is much longer than in the case of impact driving. The pile is kept in an oscillating motion during the entire vibratory driving process. It is generally recognized that vibratory driving is most efficient in coarse-grained (frictional) soil and less efficient in fine-grained (cohesive) soil.

Modern vibrators are hydraulically driven, which allows continuous variation of the vibrator frequency during operation. The vertical oscillation of the vibrator is generated by counter-rotating eccentric masses. The peak value of the centrifugal force F_v acting in the vertical direction depends on the eccentric moment M_e and on the circular frequency ω (= $2\pi f$) of the rotating eccentric masses:

$$F_V = M_e \omega^2 \quad (1)$$

The centrifugal force of many modern vibrators with variable eccentric moment and frequency can be adjusted continuously during operation. The displacement amplitude s and the centrifugal force F_v determine the driving ability of the vibrator. For a vibrator suspended above the ground surface, the vertical displacement amplitude, s, (single amplitude) can be determined from the following relationship:

$$s = \frac{M_e}{m_t} \quad (2)$$

The “total dynamic mass” m_t is the sum of all the masses that need to be accelerated by the vibrator. This includes the vibrator, the pile and the vibrator clamp. Note that most equipment manufacturers express the displacement amplitude as peak-to-peak (“double”) amplitude. From Eq. (2) it can be appreciated that the displacement amplitude s is independent of the vibration frequency, f. In order to maximize the displacement amplitude, the total dynamic mass, m_t should be kept as small as possible. Vibrators with variable eccentric moment allow the machine operator to start up and shut down the vibrator at zero vibration amplitude, thus reducing the risk of vibration amplification due to resonance.

3 Amplitude-dependent pile-soil interaction

In addition to lower losses when transferring the energy to the pile and the longer length of the force wave, there are also other fundamental differences between the pile-soil interaction of impact-driven piles and that of vibratory-driven piles, as discussed in the following.

3.1 Pile shaft resistance

In the case of impact driving, it is necessary to overcome the inertia of the pile and the shear resistance along the pile-soil interface. At the end of each impact, the pile penetration slows down and static conditions return along the pile. In the case of vibratory driving, the pile is kept oscillating axially (usually vertically) during the entire driving phase. Several hypotheses have been proposed in order to explain why vibratory driving is more efficient than impact driving in coarse-grained soil. Examples of possible causes of reduced shaft friction (permanent and/or temporary) mentioned in the geotechnical literature are “rolling friction”, “liquefaction”, and “material degradation”. However, these terms are mainly descriptive and therefore difficult to quantify. A more rational concept is needed, which can be based on the cyclic forces generated during vibratory driving. Field measurements of ground vibrations during vibratory driving have shown that the vertically oscillating force creates – due to shaft friction – a horizontally oscillating force. As this horizontal force is directed away from the pile shaft, it reduces the shear resistance with each downward movement [3]. This phenomenon, which is confirmed by field measurements presented below, is believed to be the main reason for enhanced pile penetration in coarse-grained soil.

In fine-grained (cohesive) soils, shaft resistance decreases due to strain and the number of vibration cycles (remoulding) occurring when the relative displacement between the pile and the soil exceeds about 5–10 mm. The magnitude of the eccentric moment of the vibrator is therefore important for the vibratory driving of piles in cohesive soils as it determines the relative displacement between pile and soil, see Eq. (2).

3.2 Pile toe resistance

The mechanism of pile-soil interaction during vibratory driving has been studied by several investigators [4], [5], [6]. The following simplified model illustrates the motion of the pile toe during one vibration cycle [7]. Fig. 1 shows the toe force F_T versus penetration z. At (1), the pile has completed a downward motion and starts the upward re-

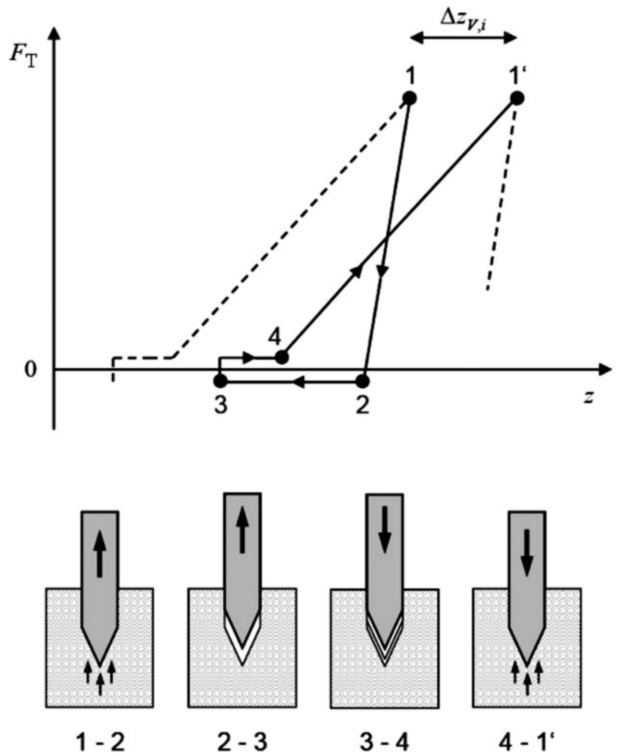

Fig. 1. Simplified model of pile toe-soil interaction during vibratory driving, after [7]
Bild 1. Vereinfachtes Model der Wechselwirkung zwischen Pfahlspitze und Boden beim Vibrationsrammen, nach [7]

bound movement of the cycle. At first, the soil below the pile toe will follow the upward movement in an elastic response. At (2), the contact force between the pile toe and the soil is reduced to zero. If the upward movement of the pile stops at (2), the pile toe will not become separated from the underlying soil. This will be the case if the displacement amplitude of the pile is small. If the upward movement of the pile continues, the pile toe separates from the soil. During this phase, a void can be generated between the soil and the pile toe, causing suction ("cavitation") between the pile toe and the soil below the toe. This phase of vibratory driving is important as this can result in remoulding and/or loosening of the soil below the pile toe. When the oscillation cycle is reversed (3), the pile again moves downward and, after some movement, regains contact with the soil (4). The toe resistance during the following cycle (1′) depends on the effect the previous vibration cycle had on the soil below the toe.

The variation in the toe resistance during vibratory driving is fundamentally different to that of impact driving. In the case of impact driving, the pile toe will remain in contact with the underlying soil. In contrast, in vibratory driving, the pile toe can separate from the underlying soil. As mentioned, the separation depends on the displacement amplitude (2)–(3) and the remoulding/suction effect during this phase of vibratory driving. If there is little or no separation between the pile toe and the underlying soil, the resistance to downward movement (penetration) will be approximately similar to that generated by impact driving. It is apparent that the vibration amplitude influences the toe/soil resistance during vibratory driving. The mode of toe penetration during vibratory and impact driving also has an effect on toe bearing. If the vibration amplitude (and therefore the toe-soil separation (2)–(3)) is small during the final phase of pile installation, soil decompression below the toe will be low. For this reason, it is advantageous to reduce the vibration (displacement) amplitude gradually over a minute or two at the end of driving toe-bearing piles, as opposed to turning off the vibrator suddenly.

3.3 Horizontal ground vibrations

A comprehensive test programme in [8] investigated whether horizontally oscillating waves occur in the soil during vibratory driving. An approx. 12 m long steel probe was vibrated at a frequency of 27 Hz into a deposit of dredged sand. The probe depth was 7 m. The vibrator used was a Müller MS 200 HF (centrifugal force: 4000 kN, max. eccentric moment: 1900 Nm) with variable frequency (0–30 Hz). Geophones were installed at different distances from the pile. Fig. 2 shows horizontal ground vibrations (velocity vs. time) measured at four geophones placed 3 m from the pile – on the ground surface and at depths of 1.65, 3.55 and 5.05 m. It is apparent that the vibration velocity is approximately the same at all levels (indicating that the vertically oscillating probe is the source of vibrations). The vibration frequency was 27 Hz, identical to the operating frequency of the vibrator. (The distance between horizontal grid-lines in Fig. 3 is 50 mm/s).

Fig. 2 shows that in coarse-grained soil, a vertically oscillating probe generates not only vertical, but also horizontal vibrations – and these can be quite strong. They manifest themselves as a horizontally oscillating wave field, which builds up horizontal stresses as a result of continuous cyclic loading. The pulsating, horizontal stresses reach their maximum at the end of each downward (and upward) cycle. The soil is pushed away from the oscillating probe, which is compressed horizontally, building up high horizontal effective stresses.

3.4 Implications of horizontal ground vibrations

During vibratory driving in coarse-grained soil, the shaft resistance of a pile will be reduced due to the horizontally oscillating stress field. The horizontal stresses are directed away from the pile and can cause arching in a cylindrical zone surrounding the pile. However, this arching effect will disappear when piles are installed in a group due to the fact that each pile driven will "disturb" the arching effect that has built up around the pile driven previously. Moreover, observations in soil compaction projects have shown that horizontal stresses will equalize to an average value over time. It is therefore difficult to estimate the shaft resistance of a vibratory driven pile.

Horizontal ground vibrations are also important when probes or piles are used for deep vibratory compaction of coarse-grained soil. The primary objective is to increase soil stiffness and strength. However, as stated above, vibratory compaction results in a permanent increase in horizontal stresses. The increase in lateral effective stress following vibratory compaction has been measured using field investigation devices, such as pressure meters [9], dilatometers [10] and cone penetrometers [11].

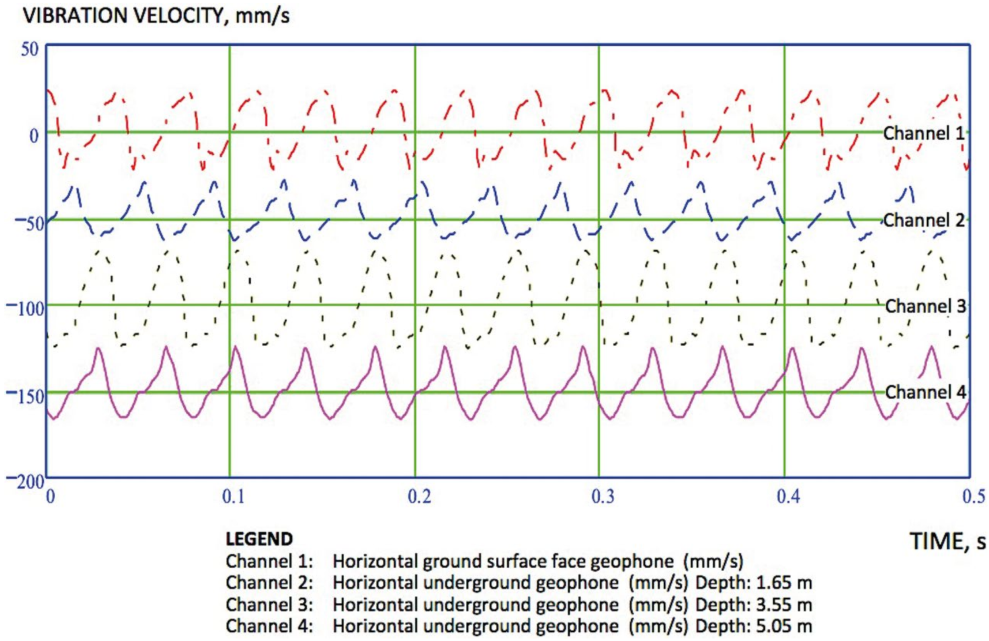

Fig. 2. Horizontal ground vibrations measured at 3 m from compaction probe oscillating at 27 Hz; penetration depth of pile = 7 m, taken from [8]
Bild 2. Horizontale Bodenschwingungen, gemessen in 3 m Abstand von der Verdichtungsstange, bei einer Schwingungsfrequenz von 27 Hz. Die Eindringtiefe des Pfahls war 7 m, nach [8]

A permanent increase in horizontal stress due to vibratory compaction by surface rollers was investigated in [12], which describes a significant increase in horizontal stress (up to the passive earth stress). Ref. [13] confirms that vibratory compaction of coarse-grained soil, in a process of load application and removal, can result in a significant increase in permanent residual horizontal stress. The researchers developed a hysteric model for multi-cyclic K_0-loading that can be used to predict the increase in lateral effective stress. They concluded that compaction represents a form of preconsolidation, similar to the application and removal of a superficial static surcharge stress.

Fig. 3 shows a representative CPT from the test site. The left diagram shows the cone stress and the right diagram the sleeve friction, before and after soil densification. The vibratory compaction resulted in a permanent increase in cone stress and sleeve friction. The cone stress

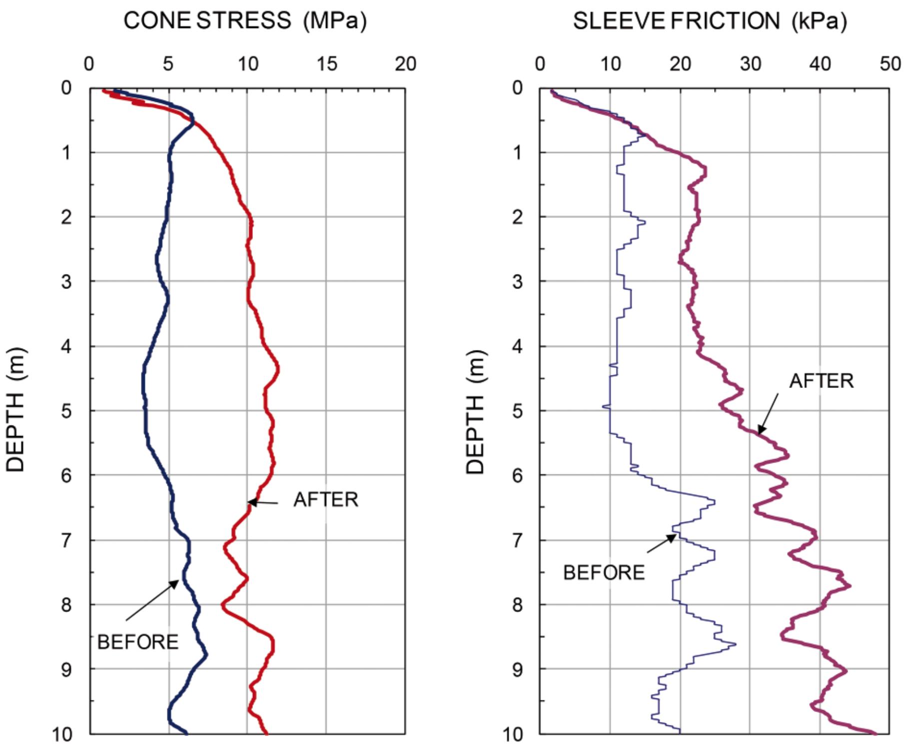

Fig. 3. Filtered average cone stress and sleeve friction values before and after compaction, taken from [3]
Bild 3. Gefilterte Mittelwerte des Spitzenwiderstandes und der Mantelreibung, vor und nach der Verdichtung, von [3]

was raised from approx. 5 to 10 MPa and the sleeve friction, which is directly affected by horizontal effective stress, increased from 10 kPa to a maximum of 45 kPa, corresponding to a factor of 2 to 3. Such a change in sleeve friction is related to a change in horizontal effective stress.

The implications of a permanent increase in horizontal stress due to vibratory compaction are important as these change the coefficient of lateral earth pressure at rest. A concept was presented in [11] to show how the preconsolidation margin (or overconsolidation ratio, *OCR*) can be determined from CPT soundings.

4 Frequency-dependent pile-soil interaction

One important parameter that affects the penetration resistance during pile driving and vibratory compaction is the operating frequency of the vibrator. Experience from a large number of soil compaction projects – and especially where the resonance compaction system was used – demonstrates the importance of vibration frequency when compacting coarse-grained soil. In order to analyse theoretically the interaction between a vertically oscillating element in an elastic medium, ref. [14] used a two-degrees-of-freedom (2DOF) system. The mass of a pile m_p interacts with the surrounding soil m_s through springs k_T and k_M and dampers d_T and d_M, as shown in Fig. 4. Note, however, that the objective of this analysis was to study the interaction of a pile with the soil at resonance, i.e. when the pile is vibrating in phase with the surrounding soil, and that this model is not applicable to the pile penetration phase (elasto-plastic conditions). Different conditions apply during pile penetration.

The resistance of the soil along the pile consists of two components: pile shaft resistance and pile toe resistance. The system denoted "*P*" models the soil in contact with the pile. The system denoted "*S*" models the soil, lying further away but still participating in the dynamic action. The pile is regarded as rigid because the shear modulus of a steel or concrete element is much higher than that of the soil (by about 10^3). The total resistance between the soil and the pile (spring k_P and damper d_P) then consists of the sum of the shaft (mantle) and toe resistances k_M and k_T respectively. The vertical displacements of the pile and the soil are denoted u_P and u_S respectively.

$$k_P = k_T + k_M \text{ and } d_P = d_T + d_M \tag{3}$$

The force amplitude function F_{p0} can be an arbitrary function of the driving frequency $\omega(2\pi f)$. In practice, two functions dominate: a constant, frequency-independent, amplitude function and a quadratic function that is a consequence of excitation obtained from the rotating masses.

$$F_{p0} = m_e e \omega^2 \tag{4}$$

where m_e is the eccentric mass and e the eccentric distance. Two equations of motion, expressed in displacements, velocities and accelerations as functions of time, can be obtained according to Newton's second law. The theoretical model assumes an elastic response of the soil, i.e. a soil modulus (and thus wave speed) independent of strain (displacement). As will be shown below, this assumption is only correct at or near resonance, when the pile element interacts with the surrounding soil.

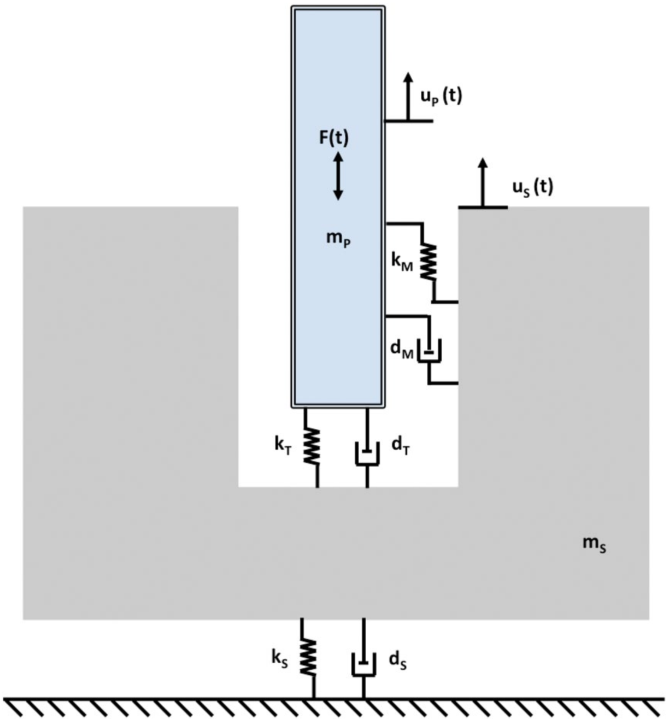

Fig. 4. One-dimensional soil model with two degrees of freedom for analysing pile-soil interaction [14]
Bild 4. Eindimensionales Bodenmodell mit zwei Freiheitsgraden zur Analyse des Zusammenwirkens von Pfahl und Boden [14]

During the penetration phase, the pile moves relative to the soil and the soil at the pile interface is in a plastic state (failure, i.e. a plastic state is, by definition, required to allow the pile to penetrate into the soil). The derivation of the theoretical solution to this pile-soil interaction problem is too complex to be included in this paper; for details see [14]. The following example illustrates the application of the theoretical model with the aim of demonstrating the influence of vibration frequency on the movement of the pile and the surrounding soil. Table 1 shows the parameters assumed in the analysis.

The first important result of the analysis is the displacement response of the vibrating pile u_P as a function of vibration frequency (real, imaginary and absolute values). For the case shown in Fig. 5, the values of eccentric moment and shear wave speed were assumed to be M_e = 10 kgm and c_S = 225 m/s. Ref. [14] showed that a clear resonance peak of the vibrator-pile-soil system occurs,

Table 1. Parameters assumed in dynamic pile-soil response analysis
Tabelle 1. Bei der dynamische Pfahl-Boden Response-Analyse angenommene Parameter

Eccentric moment	M_e	10	kgm
Total density of soil	ρ	2000	kg/m³
Poisson's ratio	ν	0.33	–
Side length of pile	b	600	mm
Mass of vibrator incl. clamp	m_{VC}	3500	kg

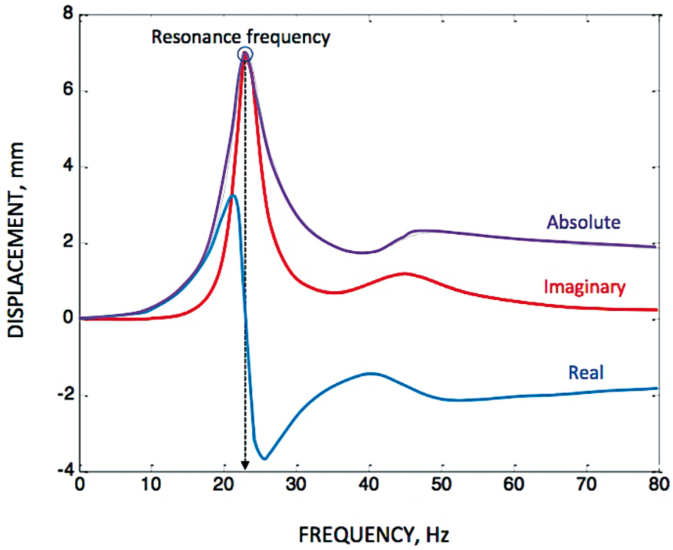

Fig. 5. Displacement amplitude u_P of a pile as a function of frequency; resonance occurs at 23 Hz
Bild 5. Verschiebungsamplitude u_P des Pfahls in Abhängigkeit von der Frequenz. Resonanz bei 23 Hz

with an amplification factor of about seven for the absolute amplitude at the resonance frequency.

The next step in the analysis was to determine the relative movement of the pile against the soil, especially in the frequency range close to resonance. Fig. 6 shows the ratio of absolute displacement of the soil and the pile u_S/u_P. This ratio can be interpreted as a transfer function of vibration energy from the pile to the surrounding soil. The important conclusion from Fig. 6 is that, below the resonance frequency, the relative displacement between the pile and the soil is very small ($u_S/u_P < 0.36$) and the pile and the soil move almost in phase, displaying only very little relative movement. In practice, it can be assumed that at and below the resonance frequency, the soil is almost elastically attached to the pile. However, when the vibration frequency increases significantly beyond the resonance frequency (by more than a factor of 1.5), the relative displacement becomes larger and reaches a peak at approximately twice the resonance frequency. It is important to note that when the relative displacement increases, the pile-soil interface will be in a plastic state, resulting in a much lower soil stiffness (and thus shear wave velocity). As a result, the pile will penetrate more efficiently at a high vibration frequency. Note that the proposed elastic model is not valid during the pile penetration phase (high frequency).

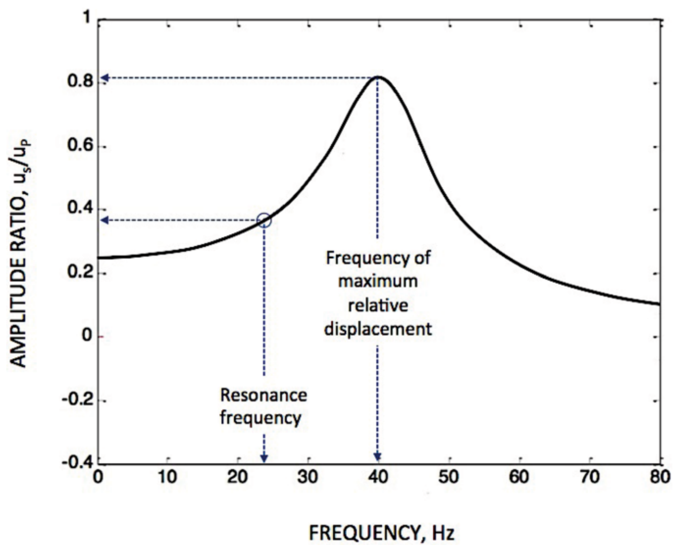

Fig. 6. Amplitude ratio of soil/pile displacement (real part) as a function of frequency. Resonance occurs at 23 Hz (see Fig. 5), whereas the maximum ratio (relative movement) occurs at about twice the resonance frequency (about 40 Hz) [14].
Bild 6. Amplitudenverhältnis der Verschiebung von Boden und Pfahl (Realteil) in Abhängigkeit der Frequenz. Resonanz entsteht bei 23 Hz (siehe Bild 5) während das maximale Verhältnis (relative Verschiebung) nahe der doppelten Resonanzfrequenz (bei etwa 40 Hz) [14]

Resonance of the vibrator-pile-soil system is a function of several parameters, with the shear wave speed (and therefore the shear modulus) being one of the most important. For most practical applications, the shear wave speed of undisturbed medium dense sand ranges between 150 and 250 m/s. However, in the presence of contiguous strong ground vibrations, the shear wave speed may reduce due to strain softening effects. For most cases, the resonance frequency is in the range of 15 to 25 Hz and decreases with increasing pile length (and ratio of vibrator to pile mass). Note that the eccentric moment does not influence the resonance frequency.

Although the theoretical analysis presented above is based on a simplified 1-D model, it captures important aspects of the vibratory driving of piles and sheet piles in soil. An important aspect of vibratory pile (or sheet pile) driving (or extraction) is that at – or close to – resonance, the penetration speed of the pile slows down dramatically as the soil and pile (sheet pile) vibrate in phase. This aspect is used in the case of resonance compaction. The following main conclusions can be drawn:

- a clear resonance peak, where pile and ground vibrations are very strong, occurs during vibratory driving,
- vertical ground vibrations reach a maximum at the resonance frequency of the vibrator-pile-soil system,
- at (and below) resonance, the relative movement between pile and soil is small, resulting in an almost static pile-soil interaction, and
- one of the most important parameters affecting resonance frequency is the shear wave speed of the soil.

5 Vibrator performance

5.1 Vibrator performance parameters

Vibrators have undergone rapid developments in terms of power, range of operating parameters (eccentric moment and frequency) and monitoring of the driving and extraction process. (Initially, before 1990, most hydraulic vibrators had a fixed eccentric moment, with a typical operating frequency of 22–30 Hz. These vibrators were used for conventional construction work, such as driving and extracting sheet piles.)

A major development in vibrator design came with the introduction of the stepwise adaptation of the eccentric moment according to specific driving requirements. Such a change to the eccentric moment is made manually by adding or removing weights. For example, if high-fre-

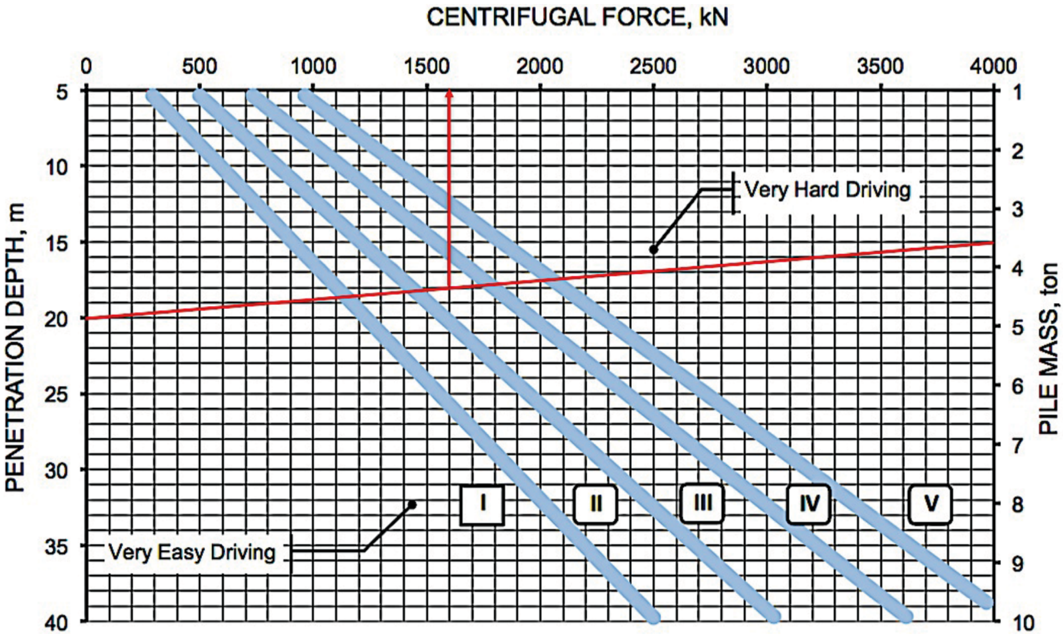

Fig. 7. Relationship between centrifugal force of vibrator, pile mass and penetration depth as a function of soil stiffness according to Table 2. Note that this diagram applies to sheet piles vibrated into granular soil
Bild 7. Beziehung zwischen der Zentrifugalkraft des Vibrators, Pfahlmasse und Eindringtiefe in Abhängigkeit von der Bodensteife entsprechend Tabelle 2. Es ist zu beachten, dass das Diagramm für Spundbohlen gilt, die in Reibungsböden einvibriert werden

quency operation is required, weights are removed on site to increase to the desired frequencies with the same centrifugal force.

The introduction of vibrators with variable frequency and amplitude allowed resonance-free starting and stopping of vibratory driving. Such vibrators allow the operating frequency and eccentric moment (and thus amplitude) to be varied according to driving requirements and soil conditions. Vibrator operation is computer-controlled and programmable.

The vibration amplitude given by vibrator manufacturers is usually in terms of double amplitude and applies to a freely suspended vibrator (without clamp and pile/sheet pile). The vibration amplitude is an important parameter and must take into account the mass of the clamping device and the pile. Note that the displacement amplitude is not affected by the operating frequency of the vibrator, see Eq. (2).

5.2 Selection of required vibrator capacity

Vibrator manufacturers have developed empirical guidelines for the selection of vibrator capacity. Fig. 7 shows the centrifugal force required to install sheet piles with varying length and mass into a soil deposit. Five soil categories (I to V) are used to describe the stiffness of the soil according to the soil classification given in Table 3. Empirical guidelines must be treated with caution and require that the user understands the limitations involved. In the example shown in Fig. 7, it is assumed that a 20 m long sheet pile (left vertical axis) with a mass of 3600 kg (right vertical axis) is to be driven into medium dense sand (soil category III) with a penetration resistance (DPH) $N_{10} \approx 12$. From Fig. 7 it can be concluded that a vibrator generating a centrifugal force of 1600 kN will be required.

An important limitation of Fig. 7 is that the toe area (cross-sectional area) of the sheet pile or pile is not included when selecting the vibrator capacity. Yet the toe resistance during vibratory driving is affected by the size of the toe area. The static cone stress q_c can be used to make a conservative estimate of the dynamic force required for the pile to penetrate into friction soils. Fig. 8 shows the centrifugal force as a function of pile toe diameter for different values of CPT cone stress q_c. For soil classification, reference is made to Table 3. For instance, a tubular steel pile with closed toe 350 mm in diameter requires a centrifugal force of approx. 1000–1500 kN in order to penetrate into medium dense soil (soil category III).

Another parameter that needs to be considered is the relative displacement between the pile and the soil. The displacement amplitude is of importance for the toe resistance in coarse-grained soil and for the shaft resistance in cohesive soils. The larger the relative displacement between the pile and the soil, the more efficient the driving process will be. The displacement amplitude depends on the total dynamic mass that must be accelerated by the vibrator and the eccentric moment. For a 20 m long pile having a mass of 3600 kg, the displacement amplitude will range between 8 mm (eccentric mass: 25 kgm^3) and

Table 2. Parameter ranges assumed in sensitivity analysis
Tabelle 2. In der Sensitivitetsanalyse angenommener Bereich

Shear wave speed	c_S	150–300	m/s
Pile length	D	10–20	m
Unit mass of pile	m_P	158	kg/m
Operating frequency	f	0–80	Hz

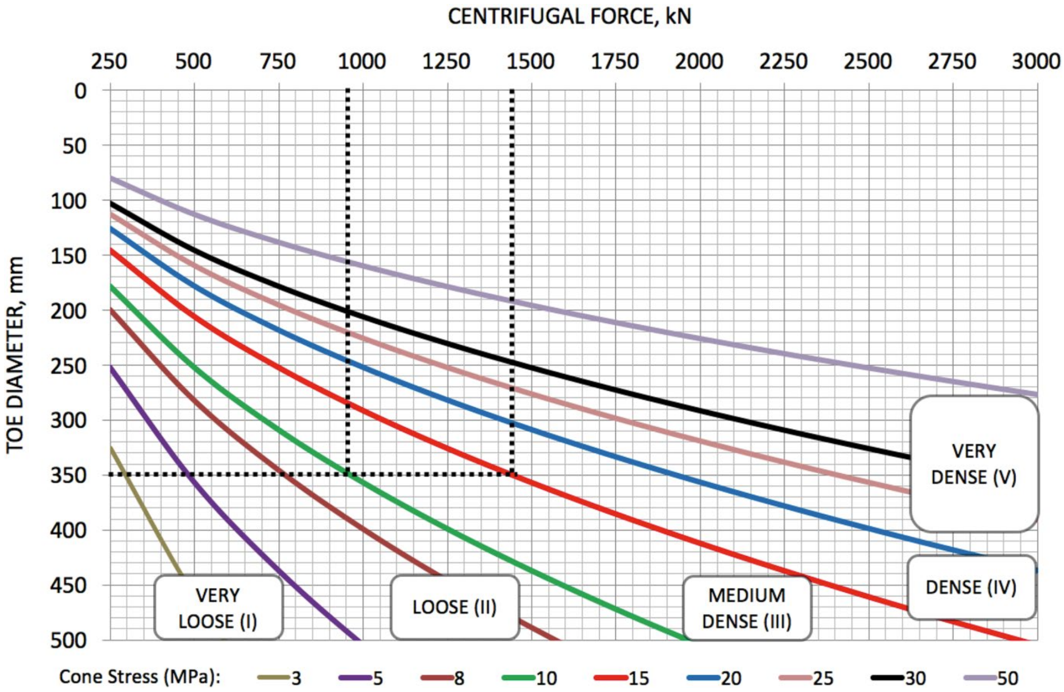

Fig. 8. Toe resistance during vibratory driving as a function of centrifugal force and pile toe diameter with the effect of shaft friction neglected and soil classification according to Table 3
Bild 8. Spitzenwiderstand beim Vibrationsrammen in Abhängigkeit der Zentrifugalkraft und Pfahldurchmesser. Der Einfluss der Mantelreibung wird vernachlässigt. Bodenklassifizierung in 5 Kategorien entsprechend Tabelle 3

Table 3. Approximate relationship between soil stiffness and driveability of piles in granular soils
Tabelle 3. Approximative Beziehung zwischen Bodenstärke/Steifigkeit und Rammbarkeit von Pfählen in Reibungsböden

Soil category		**I**	**II**	**III**	**IV**	**V**
Test type	**Unit**	**Very loose**	**Loose**	**Medium dense**	**Dense**	**Very dense**
SPT	N (blows/0.3m)	< 4	4–10	10–30	30–35	> 50
CPT	q_t (MPa)	< 5	5–10	10–15	15–20	> 20
DPH	N_{10} (blows/10 cm)	< 5	5–10	10–15	15–20	> 20

25 mm (eccentric mass: 190 kgm^3). Therefore, if the pile is to be driven into clayey soil, a large eccentric moment will result in better driving performance.

6 Monitoring of vibratory driving
6.1 Monitoring system

One important advantage of vibratory driving is that all aspects of the installation process can be monitored, controlled and documented. With modern computerized equipment, it is possible to acquire, display and record information from a range of sensors, which can be mounted on the pile, the vibrator, the power unit and the ground. Fig. 9 shows a vibratory monitoring system setup that was initially developed for resonance compaction.

When monitoring the vibratory driving of piles and sheet piles (or compaction probes), it is desirable to record the following parameters:
- Position of pile/sheet pile
- Time of recording (at least one reading per second)
- Depth of sheet pile during penetration or extraction (penetration speed)
- Operating frequency of vibrator
- Acceleration of vibrator
- Static force applied to pile (pushing or lifting force affecting vibrator weight)
- Hydraulic pressure of vibrator/power pack
- Vibration velocity on ground (geophones or accelerometers)

Ground vibrations can be recorded using geophones (Fig. 9a) and be displayed to the machine operator during vibratory driving (Fig. 9b).

Monitoring the vibratory driving process and the response of the ground and/or adjacent structures is an important aspect of modern vibratory work. For instance, in the case of vibratory driving in the vicinity of vibration-sensitive buildings or equipment, a computer-operated system can be used to control the maximum vibration intensity in order to ensure that specified limit values are not exceeded. Moreover, when vibrators are used for deep vibratory compaction, the vibration measurements can be used to guide the operator and ensure that maximum transfer of vibration energy to the surrounding soil is gen-

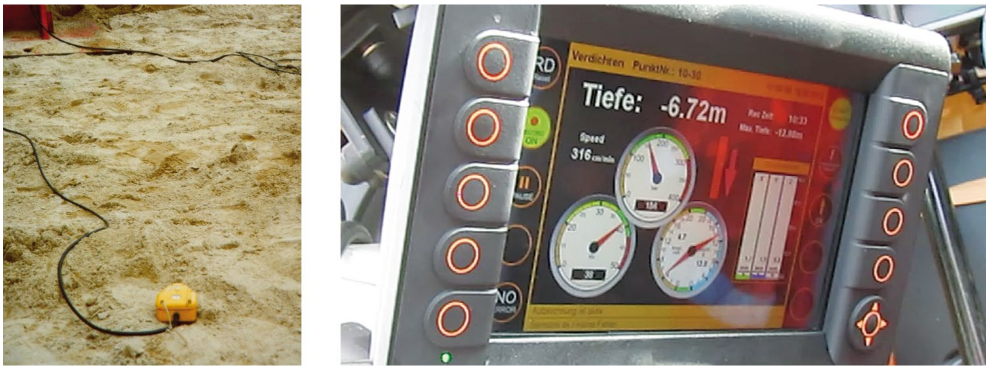

Fig. 9. Monitoring of ground vibrations and display of measurements for machine operator
Bild 9. Überwachung der Bodenerschütterung und Anzeige der Messergebnisse für den Maschinenführer

erated by the vibrator/probe system, e.g. when using the resonance compaction system.

6.2 Ground vibrations during resonance compaction

The monitoring system described above has been used on a large number of projects where resonance was used to increase compaction efficiency in coarse-grained soil. In the following case history, a tubular steel pile (with bottom plate) was vibrated into the ground using a Müller MS100 variable-frequency vibrator. The performance of the vibratory driving system (hydraulic pressure, vibration frequency, pile penetration) was monitored together with the vertical and horizontal ground vibration velocities by a geophone placed 4 m away from the compaction point. The soil profile consisted of medium to dense sand, which was loose to medium dense prior to compaction. During the initial probe penetration phase, the compaction probe was vibrated at a high frequency (35–39 Hz). During the compaction phase, the vibration frequency was gradually lowered until resonance and maximum vibration amplification was thus achieved (about 15 Hz). The vertical vibration response of the ground during penetration and compaction is shown in Fig. 10. The vertical vibration velocity at the ground surface during penetration varied between 0 and 3 mm/s, with some peak values around 6 mm/s. At resonance (about 12–16 Hz), the vertical vibration velocity increased significantly, with maximum values of 20 mm/s. The vibration amplification was about 5 to 10. After the driving performance test, the pile was extracted at a high frequency (35–38 Hz).

The shape of the measured ground vibration velocity as a function of vibration frequency is in good agreement with the theoretical vibration response (displacement amplitude), as was shown in Fig. 6. Resonance cannot occur in the horizontal direction. Field monitoring of ground

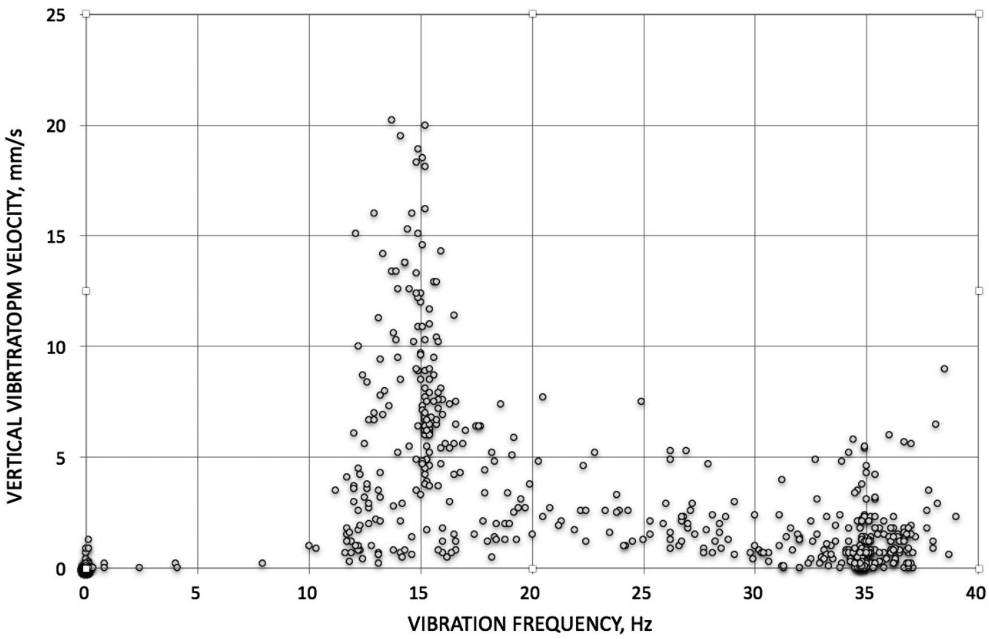

Fig. 10. Vertical vibration velocity measured at 4 m from vertically oscillating compaction probe during penetration (35–39 Hz) and compaction (12–18 Hz)
Bild 10. Vertikale Vibrationsgeschwindigkeit, gemessen in 4 m Abstand von der vertikal schwingenden Verdichtungsbohle währen Eindringen (35–39 Hz) und der Verdichtungsphase (12–18 Hz)

vibrations can provide valuable information regarding the transfer of driving energy from the pile to the soil. It is apparent that efficient pile penetration occurs when the vibration frequency is significantly higher than the resonance frequency of the vibrator-pile-soil system. On the other hand, the transfer of vibration energy, and therefore the compaction effect, is enhanced when the vibrator is operated at the resonance frequency.

7 Prediction of vibratory driving performance

When piles or sheet piles are to be installed with vibratory driving equipment, the selection of the equipment and the installation process must be based on sound information obtained from geotechnical investigations. Having to replace an unsuitable vibrator will not only result in project delays and incur additional costs, but, under unfavourable conditions, using an unsuitable vibrator can also produce damaging ground vibrations. The following section outlines a proposal for how the required vibrator capacity can be estimated based on soils information that includes records of CPT soundings and results of field trials. Using this proposed concept, it is possible to develop a correlation between penetration resistance and pile penetration speed for different vibrator types and pile sizes.

7.1 Predicting the performance of vibratory driving from penetration tests

Rational design of a vibratory driving project requires site information that includes a well-established soil profile with soil description. The most reliable geotechnical information can be obtained from a continuous record of soil layering and density, such as that provided by CPTU sounding. In Europe, heavy dynamic probing (DPH) is frequently used for assessing pile driving resistance.

Unless past experience is available from vibratory driving in similar geology with comparable equipment, field trials are the best way of estimating the vibratory driving resistance of piles or sheet piles. During the driving test, it is important that the vibrator rests on the pile and is not held back by the machine operator, which would affect the penetration speed. The following vibrator performance parameters should be measured:
- eccentric moment,
- vibration frequency,
- centrifugal force, and
- displacement amplitude.

It is also important to use measurements to verify the displacement amplitude of the vibrator-pile system prior to driving. The intensity of ground vibrations adjacent to the pile should also be recorded using a geophone or accelerometer. As the vibrator operating frequency f (Hz or rpm) is known, it is possible to convert the measured pile penetration speed v (cm/min) into an equivalent number of penetration cycles c_e per depth interval (cycles/cm):

$$c_e = \frac{f}{V} \quad (5)$$

The number of penetration cycles can now be correlated to the driving resistance of the penetrometer (DPH). This concept is illustrated in the following example. A sheet pile is driven by a vibrator, operating at 40 Hz, into a sand deposit consisting of several layers with variable DPH penetration resistance (blows/20 cm), as shown in Fig. 11. The pile penetration speed (cm/min) was measured during test driving as a function of depth, see Fig. 12. It is now possible to convert the penetration speed from Fig. 12 into an equivalent number of vibration cycles per depth interval, as shown in Fig. 13. The final step is to prepare a correlation between the penetration resistance from the penetration test and the number of penetration cycles determined from the vibratory driving test, which is shown in Fig. 14. It is possible to develop a database for

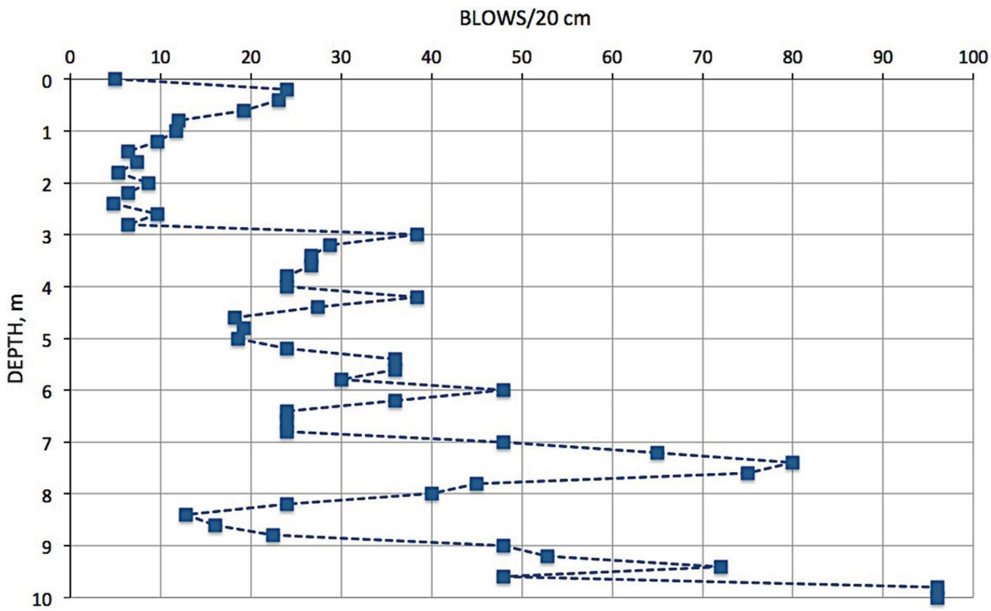

Fig. 11. Penetration resistance measured by heavy dynamic penetrometer (DPH)
Bild 11. Eindringwiderstand gemessen mit der schweren Rammsonde (DPH)

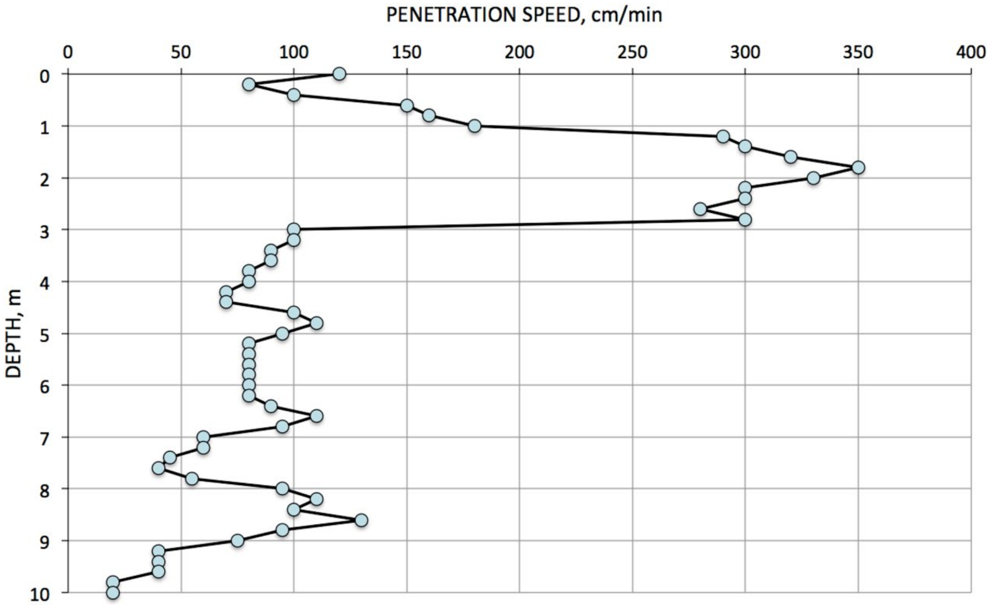

Fig. 12. Sheet pile penetration speed at constant vibration frequency (40 Hz) as a function of depth
Bild 12. Eindringgeschwindigkeit der Spundbohle bei konstanter Vibrationsfrequenz (40 Hz) in Abhängigkeit der Eindringtiefe

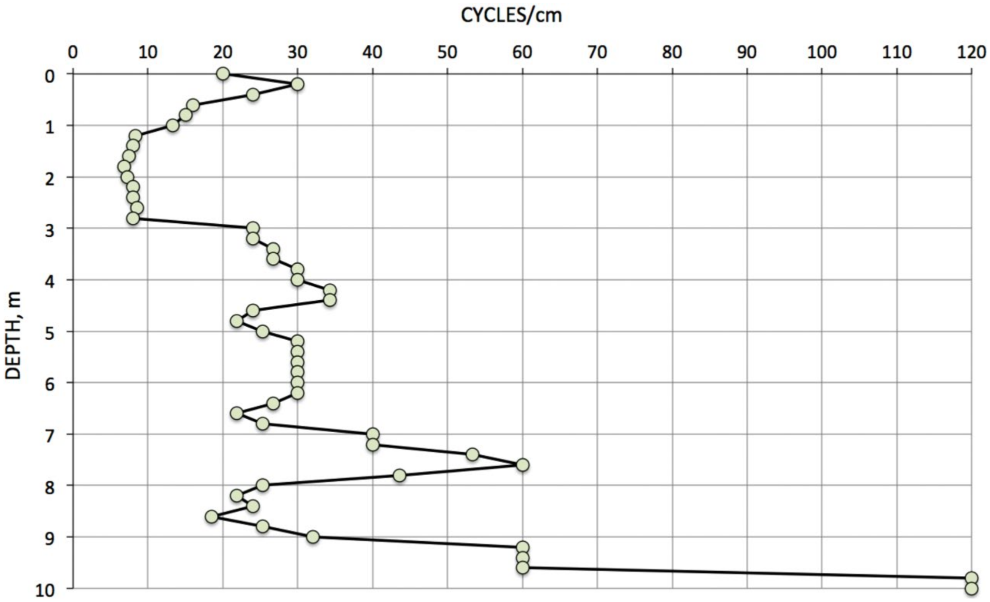

Fig. 13. Number of vibration cycles per depth interval (m) during vibratory driving of a sheet pile at 40 Hz, see Fig. 12
Bild 13. Anzahl von Schwingungszyklen pro Tiefenintervall (m) beim Vibrationsrammen der Spundbohle mit 40 Hz, vgl. Bild 12

different vibrators and pile/sheet pile sizes upon which a more reliable driveability prediction concept can be based.

8 Case history

A detailed field study of vibratory and impact driving of piles can be found in [7]. Field trials were carried out on a test site of the Institute for Technology and Management in Construction, Karlsruhe. The soil consisted of gravelly sand of variable density. The groundwater level was located at a depth of 5.4 m. A representative penetration test (DPL) is shown in Fig. 15. The sandy, gravelly soil was classified as loose to medium dense.

A 9.5 m long sheet pile, type Peiner PSp 370 with a mass of 1159 kg (122 kg/m), was suspended from a crane (“free riding”) and vibrated into the ground. During the initial driving phase (3.5 m), the vibrator had to be guided by the machine operator, which lessened the influence of the vibrator weight. Therefore, measurements before the vibrator was truly fully resting on the sheet pile are excluded from the analysis. The perimeter of the sheet pile was 2.25 m with a steel cross-sectional area of 155 cm^2. The sheet pile was installed by a vibrator (MS-10 HFV) with variable vibration amplitude and variable vibration frequency, see Table 4. Tests were performed at three vibration frequencies (25, 30 and 40 Hz), which corresponded to dynamic forces of 247, 355 and 600 kN respectively.

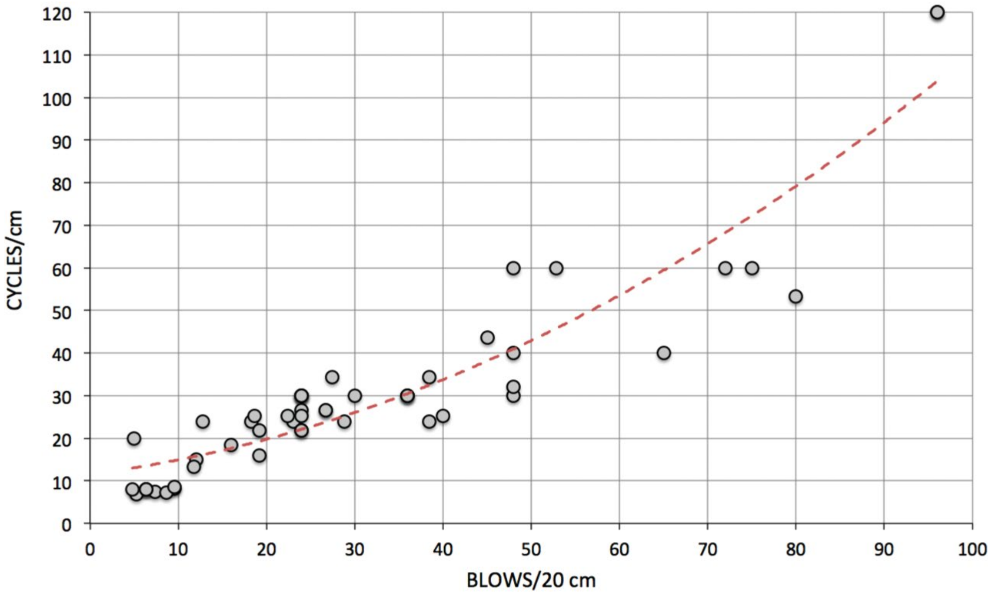

Fig. 14. Correlation between penetration resistance and number of vibration cycles per 20 cm at a vibration frequency of 40 Hz. The dynamic toe stress (neglecting shaft resistance) was 100 MPa
Bild 14. Korrelation zwischen Eindringwiderstand der Sonde und Anzahl von Vibrationszyklen per 20 cm bei einer Schwingfrequenz von 40 Hz. Die dynamische Spannung an der Pfahlspitze (ohne Mantelwiderstand) ist 100 MPa

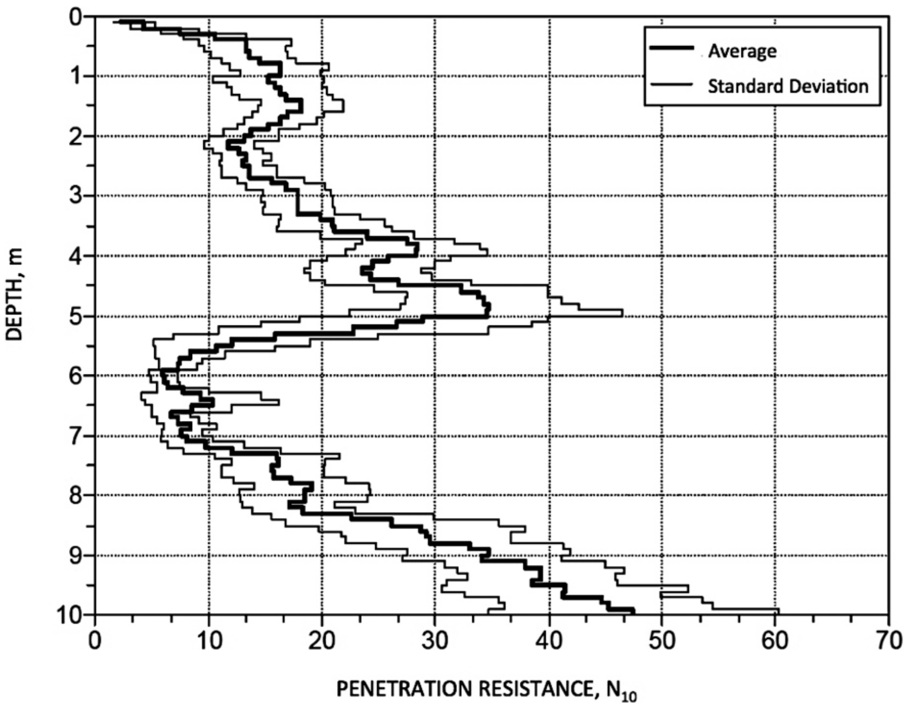

Fig. 15. Typical dynamic penetration test, light dynamic penetrometer (DPL), taken from [7]. The standard for DPL is to record the penetration as blows per 10 cm penetration
Bild 15. Typischer Rammversuch mit leichter Rammsonde (DPL), nach [7]. Der Standard für die DPL ist, die Anzahl von Schlägen pro 10 cm Eindringtiefe zu messen

The dynamic toe resistances were 16, 23 and 39 MPa respectively. The calculated displacement amplitude was 7 mm.

Different types of driving test are described in [7]. The present paper discusses only the results regarding how vibration frequency influences sheet pile penetration speed. Fig. 16 shows the penetration speed of the 9.5 m long sheet pile vibrated at three frequencies (duplicate tests for each frequency). Initially, it was intended to drive the sheet pile at 20, 30 and 40 Hz. Note that at 20 Hz, the penetration speed was very low and the anticipated penetration depth of 8 m could not be reached. In order to improve penetration, the vibration frequency was increased to 30–40 Hz. The dependence of the penetration speed on the vibration frequency is apparent: the higher the vibration frequency, the higher is the sheet pile penetration speed. Ref. [7] contains a remark that the sheet pile could not be driven at a frequency of 20 Hz.

Table 4. Performance parameters of MS-10 HVF vibrator
Tabelle 4. Leistungseigenschaften des Vibrators MS-10 HVF

Type	Definition	Magnitude	Unit
Centrifugal force	F (max.)	610	kN
Eccentric moment	M_e	0–10	kgm
Operating frequency	f	2358	rpm
Operating frequency	f	39	Hz
Dynamic mass of vibrator	M_V	1700	kg
Mass of clamping device	m_c	770	kg
Total mass of vibrator	m_V	2300	kg
Vibrator amplitude	s	11.8	mm

The penetration resistance from DPL can be correlated with the number of vibration cycles of the penetrating sheet pile, see Eq. (5). Fig. 17 shows the number of vibration cycles for 1 cm penetration as a function of the N_{10}-penetration resistance for the light dynamic penetrometer (DPL). It is apparent that there is a correlation between the N_{10}-penetration resistance and the measured number of vibration cycles per cm. Considering the uncertainty of measured values, the correlation is surprisingly good.

9 Effects of vibratory pile driving

If properly planned and executed, vibratory driving of piles and sheet piles can be carried out with minimum environmental effects such as ground vibrations, noise and soil disturbance. As has been mentioned above, the operating frequency of the vibrator has a strong influence on vibrations emitted from the vibrating pile. The highest risk of ground vibrations occurs when the vibrator is operated at the resonance frequency of the vibrator-pile-soil system. At this frequency, the penetration speed drops. This aspect is exploited for the deep vibratory compaction of granular soils [11].

9.1 Settlement due to ground vibrations

The first step is to determine vibration limits below which the risk of settlement is negligible. It is possible to determine critical vibration levels, which are based on the shear strain level generated by ground vibrations. When vibrations pass through a material, strains are induced. The strain ε caused by the propagation of a compression wave (P-wave) can be determined from Eq. (6) if the particle velocity v_P measured in the direction of wave propagation and the wave speed c_P are known.

$$\varepsilon = \frac{V_P}{C_P} \tag{6}$$

Similarly, as shown in Eq. (7), the shear strain γ can be calculated by dividing the particle velocity measured perpendicular to the direction of wave propagation by the shear wave speed c_S:

$$\gamma = \frac{V_S}{c_S} \tag{7}$$

Rearrangement of the soil particles is unlikely to occur if the shear strain is below a threshold value of $\gamma_t \approx 0.001\,\%$ [2]. When this level is exceeded, the risk of particle rearrangement, and thus settlement, increases. At a shear

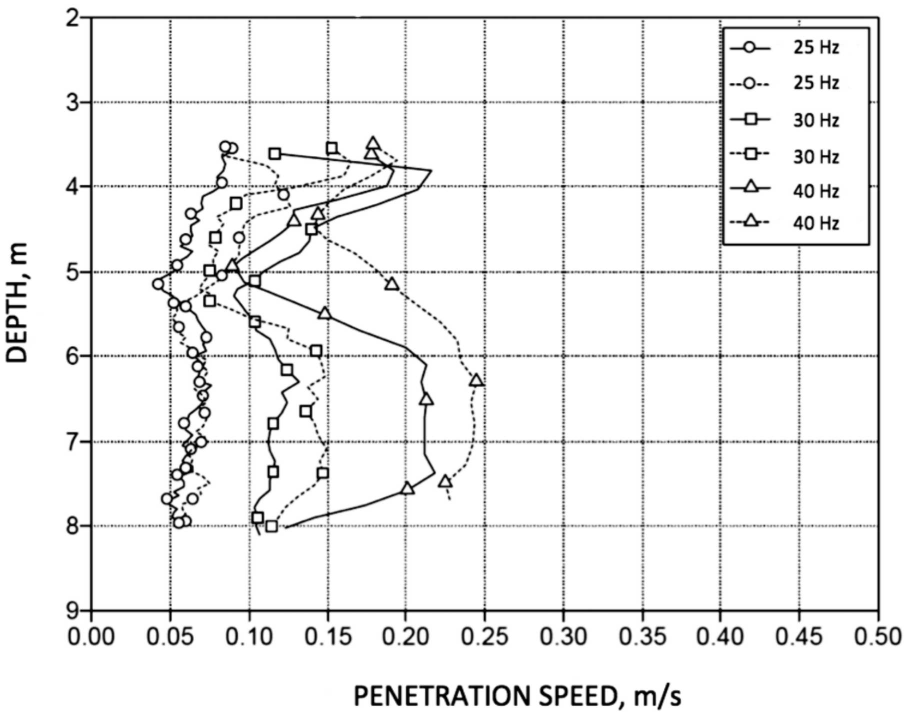

Fig. 16. Sheet pile penetration speed measured at three different vibration frequencies (two tests for each frequency), after [7]
Bild 16. Eindringgeschwindigkeit der Spundbohle, gemessen bei drei verschiedenen Vibrationsfrequenzen (zwei Versuche pro Frequenz), nach [7]

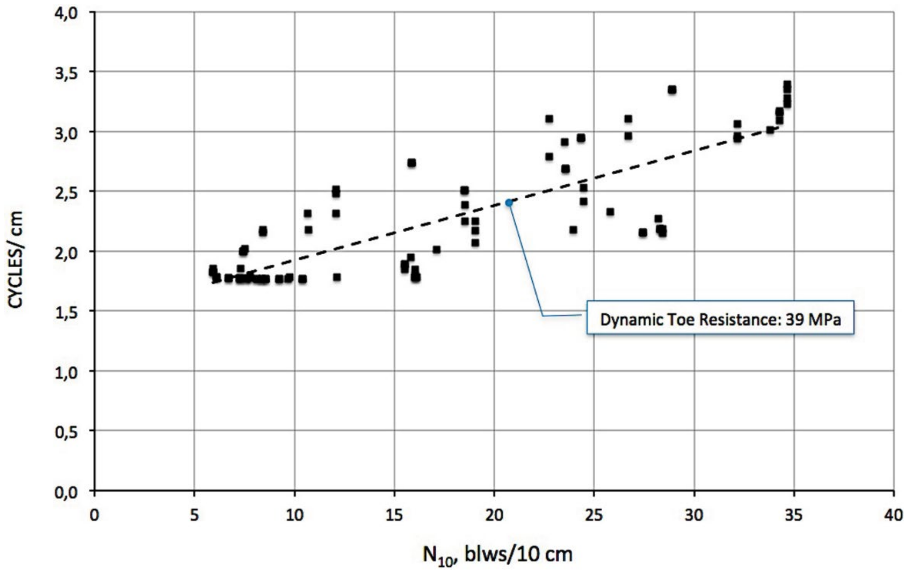

Fig. 17. Relationship between penetration resistance N_{10} and number of vibration cycles at 40 Hz operating frequency
Bild 17. Beziehung zwischen Eindringwiderstand, N10 und Anzahl von Vibrationszyklen bei einer Schwingungsfrequenz von 40 Hz

strain level of 0.01 %, vibrations can start to cause settlement, so this value should not be exceeded. Significant risk of settlement exists when the shear strain level exceeds 0.1 %. The shear wave speed decreases with increasing shear strain. Based on Eq. (7), and taking into account the reduction in shear wave speed with shear strain level, a simple chart was developed which shows the relationship between vibration velocity (particle velocity) and shear wave speed. Three different levels of shear strain in relation to the risk of settlement in sand are shown in Fig. 18.

In the following example, piles are driven in the vicinity of a building founded on medium dense sand. It is further assumed that the sand has an average shear wave speed of 200 m/s. It should be noted that the peak particle velocity, which is relevant for settlement, should be measured perpendicular to the direction of vibration propagation. Using Fig. 18 it is now possible to determine the three damage threshold levels: no risk (0.001 % shear strain) = 2 mm/s, low risk (0.01 % shear strain) = 20 mm/s and high risk (0.1 % shear strain) = 75 mm/s. In practice, a planning engineer can call for pile driving tests, where the expected vibration levels can be determined as a function of pile penetration depth and at different distances. This information can be used to assess the risk level with respect to settlement in the sand. If the predicted vibration level exceeds the "low risk" level, a detailed monitoring programme should be implemented. This simple example illustrates that it is possible to assess the risk of settlement when sandy soil is subjected to ground vibrations. Of course, a more detailed analysis can be performed which also takes into account other important factors such as number of vibration cycles etc. However, for many practical purposes, a simple assessment of the settlement risk in combination with field monitoring will suffice.

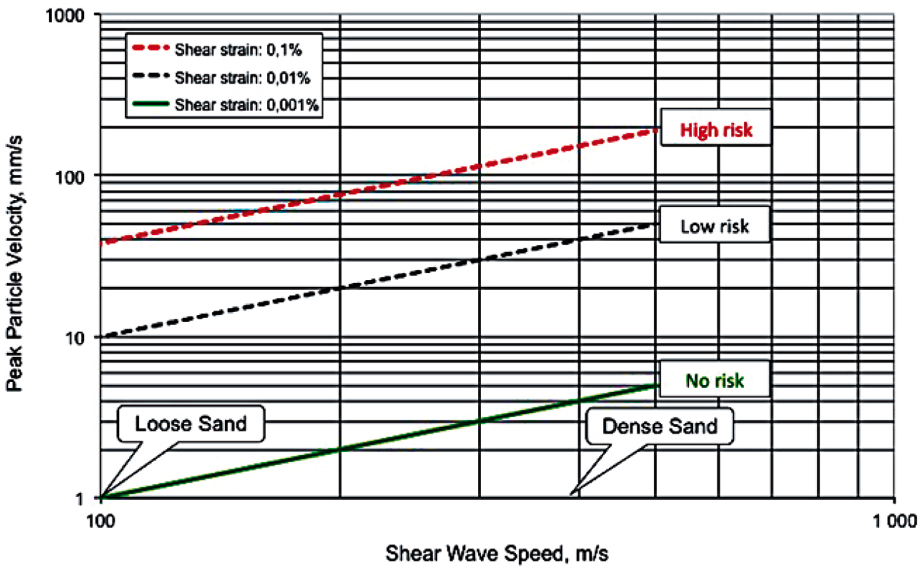

Fig. 18. Assessment of settlement risk in sand as a function of shear wave speed for different shear strain levels [2]
Bild 18. Bewertung des Setzungsrisikos in Sand in Abhängigkeit von Scherwellenschnelle für verschieden Schubdehnung [2]

9.2 Estimation of settlement adjacent to a pile driven in sand

Vibrations that are caused by driving piles into dry or permeable soils can cause settlement. The magnitude of the settlement depends on several factors, such as soil type and stratification, groundwater conditions (degree of saturation), pile type and method of pile installation (driving energy). For estimating settlement in a homogeneous sand deposit adjacent to a single pile, ref. [2] proposed a concept that is illustrated in Fig. 19. It is assumed that the most significant densification due to pile driving occurs within a zone corresponding to three pile diameters around the pile being driven. The volume reduction resulting from ground vibrations will cause significant settlements in a cone with an inclination of 2(V):1(H), with its apex at a depth of six pile diameters below the pile toe. Thus, the settlement trough will extend a distance of 3D + L/2 from the centre of the pile, with maximum settlement at the centre of the pile. Maximum and average settlement can be estimated for an appropriate value of the soil compression factor α by using the following relationships:

$$s_{max} = \alpha(L + 6D); \quad s_{av} = \frac{\alpha(L + 6D)}{3} \qquad (8)$$

Table 5 shows compression factors based on experience gained from soil compaction projects and which are applicable to driving in very loose to very dense sand. The intensity of ground vibrations can be assessed based on the vibration levels indicated in Fig. 18.

Let us assume that a concrete pile with diameter D = 0.3 m and effective pile length L = 10 m is to be installed in a deposit of medium dense sand. The pile is driven using an impact hammer and pile penetration is normal (and assuming there are no stiff layers requiring high driving energy). The compression value for medium dense sand and average driving energy according to Table 1 is α = 0.010. The maximum settlement adjacent to the pile and the average surface settlement of the cone are 118 and 39 mm respectively. The radius of the settlement cone of the ground surface footprint is estimated to be 5.9 m, resulting in an average surface inclination of 1:50 (0.118/5.90).

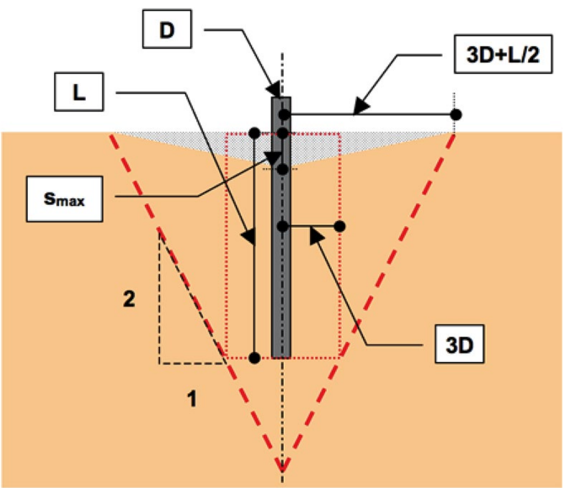

Fig. 19. Concept for estimating settlements adjacent to a single pile in homogeneous sand [2]
Bild 19. Konzept zur Abschätzung von Setzung nahe eines Pfahls in homogenem Boden [2]

Table 5. Compression factor for sand based on soil density and level of driving energy
Tabelle 5. Kompressionsfaktor, α für Sand ausgehend von der Bodendichte und der Rammenergie

Ground vibrations:	**Low**	**Medium**	**High**
Soil density	**Compression factor α**		
Very loose	0.02	0.03	0.04
Loose	0.01	0.02	0.03
Medium	0.005	0.01	0.02
Dense	0.00	0.005	0.01
Very dense	0.00	0.00	0.005

10 Conclusions

Vibratory driving of piles and sheet piles is a commonly used method, especially for the installation of foundation piles and sheet piles. The most important vibrator parameters for vibratory driving are the centrifugal force, the vibration frequency and the eccentric moment.

The toe resistance during vibratory driving is approximately similar to that occurring during impact driving. The strength and stiffness of the soil below the pile toe will be affected by the movement (displacement) of the pile toe and the number of vibration cycles. However, when the pile toe separates from the soil below during each vibration cycle, this will affect the pile toe response.

Compared with impact driving, the shaft resistance along the pile is fundamentally different during vibratory driving. In coarse-grained soils, vibratory driving of the pile shaft generates an oscillating wave field that exerts horizontal stresses directed away from the pile shaft. This pulsating wave field causes a reduction in horizontal stress acting along the pile shaft and can explain why vibratory driving of piles and sheet piles is efficient in coarse-grained soils.

The horizontal stress oscillations also cause a permanent increase in horizontal effective stresses and thus preconsolidation of the soil adjacent to the pile. This effect is important for vibratory compaction of coarse-grained soils but is generally not appreciated.

In fine-grained soils, it is important that the displacement amplitude of the sheet pile (and thus the eccentric moment of the vibrator) is large enough to overcome the adhesion resistance along the pile.

The vibrator-pile-soil interaction is a function of the operating frequency in relation to the resonance vibration frequency of the vibrator-pile-soil system. The theoretical analysis introduced here can be used to estimate the resonance frequency. The most important parameters governing the resonance frequency are the stiffness (shear wave speed) of the soil and the mass of the vibrator and the sheet pile. The eccentric moment does not affect the resonance frequency.

At resonance, the sheet pile oscillates in phase with the surrounding soil, i.e. the relative displacement between pile and soil is very small. Static friction exists along the pile-soil interface, which enhances the transfer of vibration energy to the soil. This effect is beneficial during vibratory compaction but reduces penetration speed and can cause vibration problems during pile or sheet pile installation. With increasing vibration frequency, the relative displacement between pile and soil increases, resulting in a reduction in shaft friction. Therefore, piles should be vibrated at a frequency of at least 1.5 times the system resonance frequency in order to achieve efficient pile penetration and minimize vibration emissions.

The vibratory compaction effect can be enhanced by operating the vibrator at the resonance frequency of the vibrator-probe-soil system. This concept is exploited in the resonance compaction method. At resonance, the vertical vibration velocity is about 5 to 10 times larger than at the maximum vibration frequency. Horizontal ground vibrations are significantly lower.

Empirical rules can be used to assess the required vibrator capacity (centrifugal force) for sheet pile driving in granular soil. However, a more reliable concept is to predict the driveability of piles or sheet piles based on field trials. A method is described which makes it possible to correlate the penetration resistance from dynamic penetration tests to the number of vibration cycles required for installing a pile or sheet pile. A case history is presented which demonstrates the practical application of the concept and shows that, for granular soils, a correlation can be developed between penetration resistance and number of vibration cycles. Based on this information, which can be obtained either from field trials or experience gained during past projects, it is possible to determine the penetration speed during vibratory driving. This information can be a valuable guide when selecting the vibratory driving equipment required and predicting the duration of vibratory installation of piles or sheet piles.

Vibratory driving can be an environmentally friendly pile installation method, provided that resonance effects are avoided. It is possible to determine the risk of settlement in granular soils based on the shear strain level. An empirical concept is proposed to estimate the extent of settlement adjacent to a pile.

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